How to Encourage Students Learning Mathematics Themselves:

"Double Class—Real and Virtual Class"

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February 2010

Summary

1. Background

At present, curriculum reform in Basic Education in China is at a stage where further explorations are needed. While curriculum standards require students to cultivate their abilities in autonomous learning, cooperative learning and exploratory learning, many problems exist in teaching practices and the solutions to these problems are usually superficial. The teachers and students devote efforts and cherish every second to work them out, but their ultimate goal is just earning a higher score and the method is simply doing more homework. Although the behavior of pursuing higher scores is admirable, we still need another consideration if it is essential for the students to devote so much time on their homework.

Mathematics education of China faces great challenges. They are how to encourage students learning mathematics themselves, to help the students to boost their interests in mathematics and to raise their confidence and ability to learn. Hence we start to think of new ways.

2. Two Successful Cases

2.1 R. Moore Teaching Method

R. Moore was a very excellent mathematician and was especially a very successful mathematics educational scientist. He had cultivated excellent mathematicians in U.S.A and had offered a successful teaching method in mathematics education, particularly "problem course", which has influenced many mathematics educational scientists and mathematicians. We think that the core idea of the Moore teaching method is to encourage students to learn mathematics themselves and during this learning process the teacher will give students effective directing. This teaching and learning method will not only help students obtain some mathematics content and results, but it is very important to improve the student's learning ability and to increase the student's learning confidence and interest. We like to use Moore's educational idea to improve our work.

2.2 Mathematical modeling Activity of China

The spread and application of a network in distance-education has provided us with some

new ideas. On the other hand, the exploration of mathematical-modeling has a history of more than 20 years in secondary education, and educators in this field have abundant experience and resources. In different cases, students experimented on mosquito-coil redesigning, the length of shoelaces, the problem of polyhedron-folding, chessboard redesigning, post-service optimization, the problem of packaging, shelter-forest problems and so on. Now we have an accumulation of more than 12,000 papers delivered by the students, some of them are recommended to attend college as a result. Meanwhile, the teachers gained a lot of experience during these practices. In the process of teaching and learning mathematical-modeling, the teachers and students are not only involved in traditional teaching, but they also have to search for different materials through all kinds of ways, and they use internet as their main tool to interact.

Mathematical modeling activity impels some changes in mathematics education. For example, mathematical modeling has already entered into the classroom in high schools; Modeling has changed "teaching" and "learning"; Modeling cultivates the awareness about asking questions and innovating spirit; Modeling has brought great changes to mathematics curriculum in China.

Conventional mathematical modeling can be divided into the following steps:

Teachers provide students with the background of problems, students collect information and identify the issue

To determine the outline of the investigation or information according to the problem,

To search on the Internet or do the appropriate investigation during their spare time.

To solve the problem independently, seek appropriate help and guidance if necessary;

To discuss and exchange results, teachers and students appreciate, question and evaluate each other.

In this process, first of all, we encourage individuals to study and think independently. On this basis, students can form a mathematical modeling group, and discuss together, inspire mutually, divide the task, and explore for the solutions.

Students like to do this activity very much. Here are some titles of students' papers:

" Optimization problems of banking counter" Chenle's group

• "To determine the appraisal index of basketball shoes" Zhang Boyang's group

• "Quantitative analysis of the value of stocks" Deng Xiaoran's group

• "Research on vehicle emissions" Liu Gezi's group

• "Optimization of computer keyboard" Zhu Chenran's group

• "Public Transport in Beijing" Wang Zihao's group

• "Mathematical problem in rope skipping" Meng Han's group

• " Functional relation between the time and date of raising flag in Tiananmen Square",Li Shuo's group

"The changing tendency of death toll reported in an earthquake" Gao Yinxiang's group

• "Forecast of arrival time of population peak in China" Ye Mai's group

• "The functional relation and determinants of applied frequency of Chinese characters" Chen Chongyao's group

• "Relationship between profits and cost of clothes, original cost, number of pieces, total profit, increased quantity of sale for cutting prices, etc." Shen Da's group

• "Charges of Telecom mobile phone's new package" Ludi's group

• "The relationship between basketball shooting angles and hit rate" Song Chang's group
• "Gold futures" Wang Kaizheng's group
• "Research the price changes of mobile phone with a function" Yueguang's group
• "Trend in New Year box office," Lu Yi's group
• "Relationship between the rotation of bicycle pedal and the distance traveled" Chen Yingjiao's group
• "Calculating bank rate" Cao Zhengwang's group
• "Out of the strange cycle" Lv Lansong's group
• "Throwing solid ball" Zhang Yubai's group
• "The best laundry program" Le Shuo's Group
• "How to shoot to enhance the hit rate" Gong Zibo's group
• "Optimal design of beverage cans," Chen Zhaochu's group
• "The relationship between the area of shadow cast by the south window and the time of the day," Wang Xueyun's group
• "Which is more suitable for you, Shenzhouxing or M-Zone?" Ye Shiqing's group
• "The principle of diminishing marginal utility in Running" Shen Sicheng's group
• "The best initial speed for vehicle's sideslip" Guo Hongtao's group
• "Aircraft bomb problem" Liliang's group
• "The number of light, angle, wattage, distance and the area illuminated" Chengnan's Group
• "Changes in Watermelon's price and forecast of future prices" Wang Qingnan's group
• "The functional relation between temperature of water in a bouilloire and the electricity consumed" Ni Zengtao's group
• "Flickering flame of fireworks in the sky and the weight of flame granule" Shen Yichen's group
• "Mathematical analysis in the Billiards" Liu Yehong's group
• "The relationship between the temperature of the asphalt road and the temperature of air near the ground" Tan Wangshu's group
• "Research on water temperature change" Wang Zhongshu's group
• "Idea from 'ink spreading'" Wang Shu-yu group
• "The relation between the rotation angle of gas stove knob and gas volume used" Yang Liqiang's group

We hope to use these experiences to make a new teaching and learning pattern to encourage students to learn mathematics themselves.

3. Project and Challenges

In 2007, the experiment of curriculum reform in ordinary senior high school in Beijing is to be launched. The experiment, which is led by the Civil Basic Education Reform Commitee, consist of 16 major research projects, all of which shall be delivered and executed individually. One of the major research projects is the "Deepening the Reform of Teaching Methods and the Scientific Applications of Information Technology" project, whose characteristic is to combine the virtual classroom online and the real classes (which is also called dual class combination module or dual class for convenience). The project is directed by the Beijing Education Association, and subjects like Chinese, history, geography, music and math are involved.

In the 1st round of experiment, subjects as Chinese, history, geography and music are involved, and routine teaching in virtual classes are combined with conventional teaching methods. The students' interests and confidence can be lifted in this way, while their abilities to question, to communicate, and to express can be improved, and their learning concept be changed dramatically. The discipline of mathematics is characterized by abstractness, logicality and its wide applicability. The students have little incentive in a conventional classroom, so many teachers wonder if they could learn autonomously in virtual classes. With much doubt we began this experiment on mathematics.

With its own characteristics, the mathematic discipline is facing tremendous pressure and challenge:

Is there a certain way that mathematics can be worked out? Can the students learn mathematics via the Internet with the help of the teachers?

What kinds of content are suitable for autonomous learning?

Is there a stable teaching model that can be spread?

4. Objectives and Thoughts

Faced with problems in curriculum reform, we laid down the objectives for the experiment.

1. To promote autonomy, raise their interest and confidence.
2. To explore a new mode that will help encourage autonomy using network platforms.
3. To build and encourage the share of superior resources.

After discussions between experts, we make our thoughts clear and it can be indicated as follows:

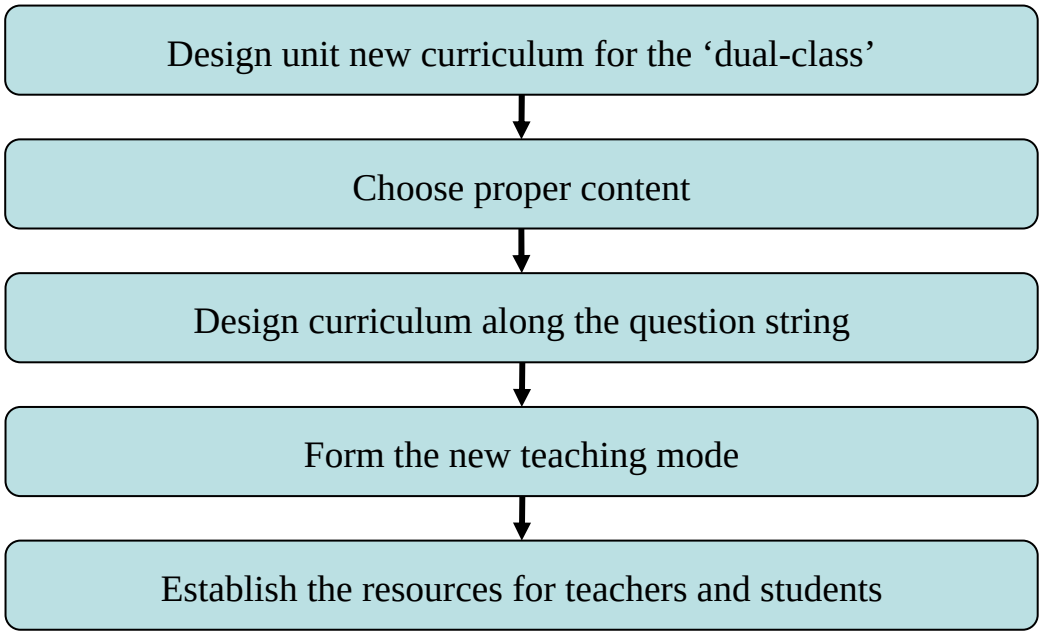


Figure 1: the train of thought for experiment

1. Design unit new curriculum for the ‘dual-class’.
   The so called unit new curriculum refers to one chapter or several chapters. It can be a

complete math activity or a special optional topic.

At present, text books used by students are actually written for teachers not for them, making it hard for students to read, and thus their requirements and needs can't be satisfied. As a result, with the design philosophy "macro-curriculum" we try our best to make learning, teaching, evaluating, and professional development improve in parallel.

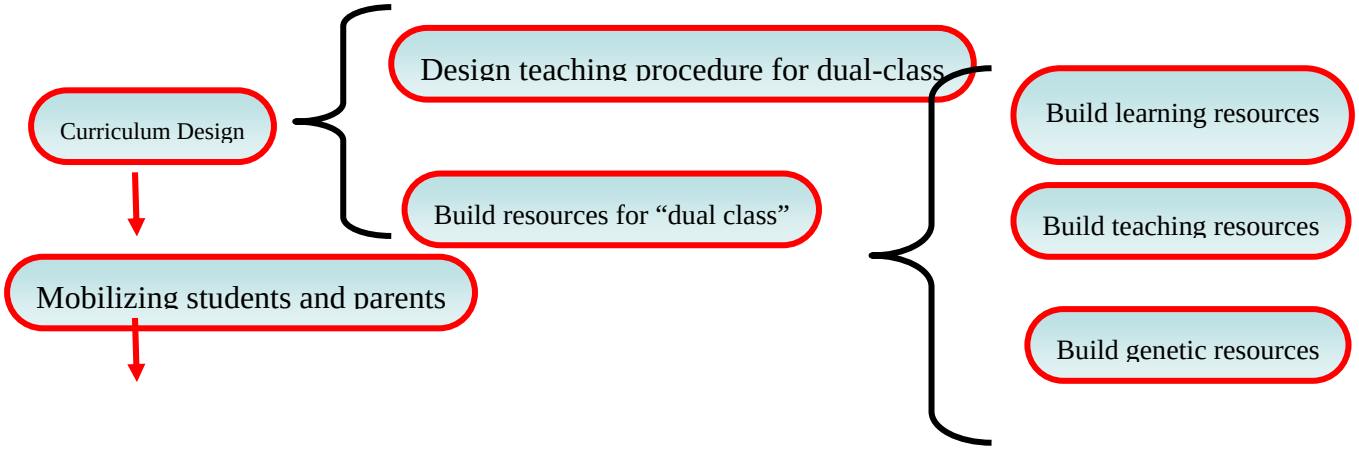
2. Choose proper content. Here are the rules for choosing:
   1) Mathematics content should be characterized by:
      - **Analogy**: From plane vector to space vector, from sine function to cosine function, form parithmetic progression to geometric progression. Conic curve.
      - **Vision:** Learn the definition and attributes of trigonometric function as well as identity transformations by virtue of unit circle. Solid geometry introduction is also included.
      - **Activity**: We offer a wide range of statistical problems to choose from. Students participate with enthusiasm, experiencing the whole process of cooperation.
      - **Summary:** Algorithm review knowledge by virtue of algorithm. (Solve inequality, dichotomy)
      - **Reason and argumentation:** Review the important theorems and definitions we have learned in the past by studying this new part.
   2) The content we choose must be complete.
3. Design curriculum along the question string.

   The big challenge is how to arrange the content we have chosen to appeal to students. Here we try our best to make some innovation. We borrow the trick in America and continue the question list.
4. Form the new teaching mode

   In the virtual class, most of the time, students learn by themselves, collaborate and discuss. Teachers just provide some interesting related applications and questions. This contrasts with the real class, where teachers are the leading role. They interpret definitions, explain main methods and guide students to discuss and solve the trouble happened in the virtual class.
5. Establish the resources for teachers and students

   In order to make our new teaching mode "Double Class-Real and Virtual Class" perfect, it is essential to develop and accumulate the lively and vivid resources which are so significant for both students and teachers.

4. Teaching mode in "Double Class-Real and Virtual Class"



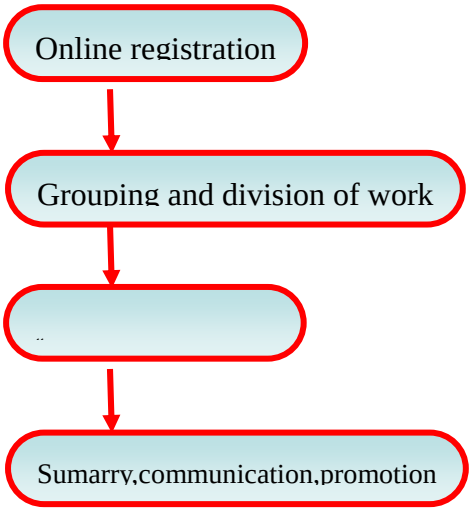


Figure 2: teaching procedure for dual-class

Our teaching mode is fixed gradually after many experiments. Figure 2 has shown all the steps which are all crucial points.

1) Curriculum Design

According to the characteristic of dual-class, the design and accumulation of learning resources and teaching resources should be based on the question string.

2) Mobilizing students and parents

For those ignorant of this new mode, some students may avoid participating in case it affects their scores. So the mobilization is necessary. Dual-class is based on the net, some students can't control themselves well, and as a result, we should chat with teachers to gain their support and trust.

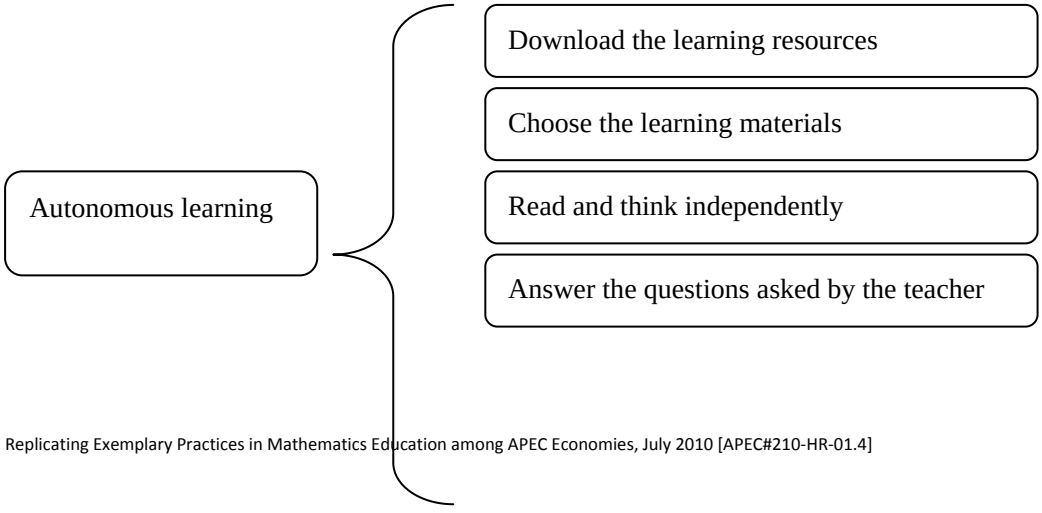
3) Online registration

Entering "virtual class", the precondition is registering online, through which teachers can supervise, manage or record everyone's status of learning. Students and teachers can discuss or communicate together freely on this platform.

4) Grouping and division of work

With their own definite task, students can easily engage respectively on their own initiative. This can cultivate their ability in collaborative learning as well.

5) Decide whether to use the "Double Class-Real and Virtual Class" according to the content.



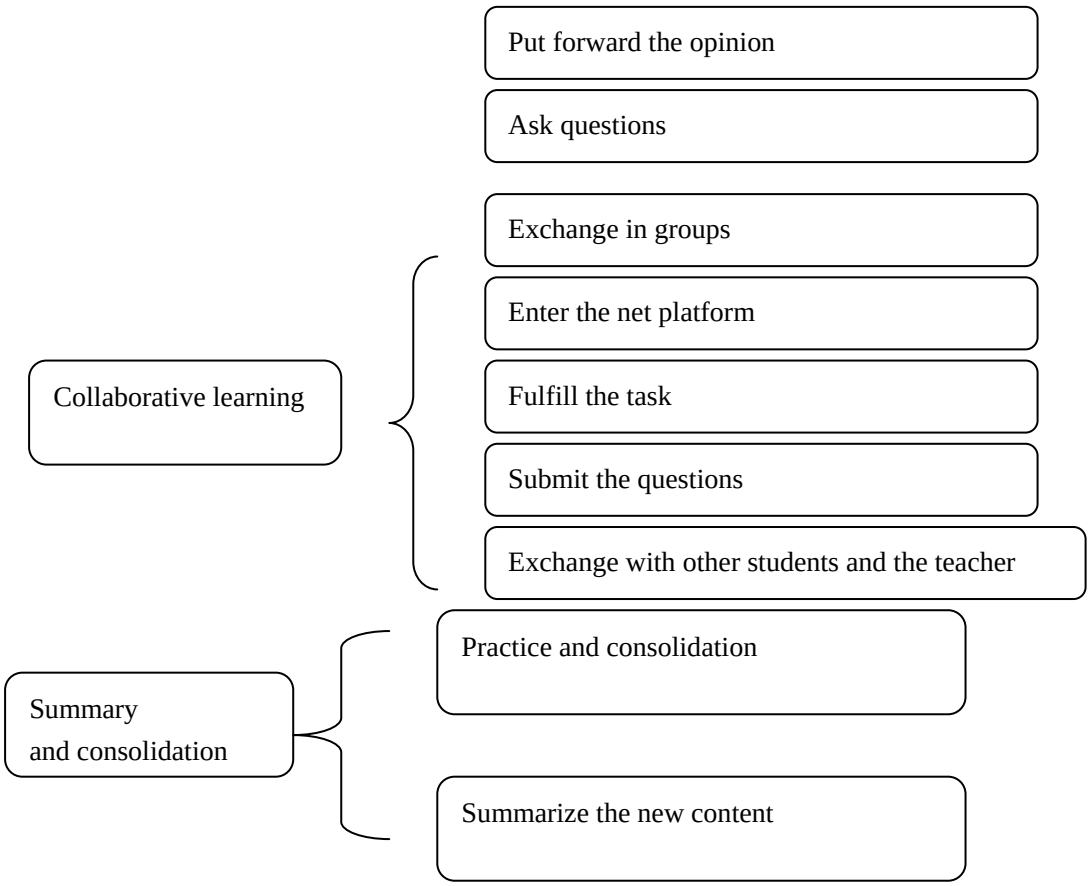


Figure3: Teaching procedure for "virtual class"

Figure 3 shows the teaching procedure in an ordinary situation. It can be adjusted if necessary according to the specific condition. It is clear that leaving sufficient time and space for students can arouse their interests in math greatly and raise their confidence. They dare to question and argue; they learn how to communicate and cooperate.

Here are some situations in the "real class"

- In the initial period, teachers let students get a general idea of what they will learn.
- Teachers explain the questions most students meet in the virtual class
- Teachers remark on the reports that students present in the real class and help them to think deeply.

5. Overview of our experiment

More than 20 schools have signed the cooperation agreement to engage in our experiment since the project has been launched by Beijing Municipal Education Commission and Beijing education society.

Table1: Schools that participated in the experiment and content involved

Number	Content	Features	Schools Experimented
1	Mathematical modeling	activity	High School Affiliated to Peking University

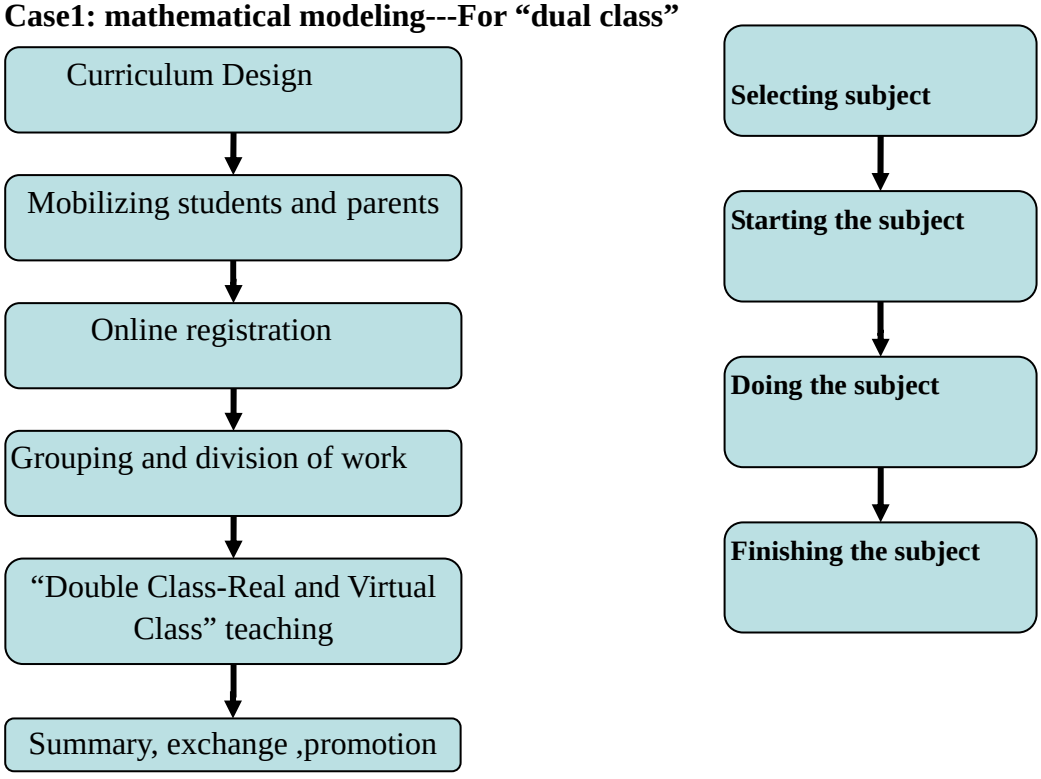
			19th High school in Beijing
			15th High School in Beijing
			Experimental School of Beijing Economic and Technological Development Zone
2	Matrices and transformations	comprehensive	1th High School in Changping
			8th High School in Beijing
			Huiwen High school
3	Solid geometry Introduction	visual	2th high school in Changping
4	Definition and application of derivative	analogy	17th High School in Beijing
			94th High School in Beijing
			Ritan High School in Beijing
5	The optional course of math history	culture	Huiwen High School
			ChenjingLun High School
6	Review and go over the knowledge of function	summary	Qianfeng High School in Changping
7	progression	analogy	Oriental Decai High School
8	conic curve	analogy	1th High School in SanliTun
			ChenjingLun High School
9	probability	other	Foreign Language High School in Chaoyang
10	inequality	other	80th High school in Beijing
11	Reason and argumentation	summary	19th High school in Beijing

Note: Some school have finished the experiment, some are still carrying it out.

6. Introduction of cases

Due to limited space, we don't show all the cases and details here, but rather only provide two cases for reference. If you would like to check out other cases, please refer to our book.

Case1: mathematical modeling---For "dual class"



Being different from other content, mathematical modeling has a unique teaching procedure--, selecting subject, starting the subject, doing the subject, finishing the subject.

Select subject: During this period, students read the related literature, review the knowledge, discuss in groups, and finally put forward the subject their group want to research.

Start the subject: After several discussions, they have a rough idea and a rough estimation about their job and then submit the report with a preliminary plan and idea on it.

Do the subject: They put the preliminary plan into practice by discussing together, asking for help online, searching for materials, choosing proper tools, writing reports, estimating and calculating. Through their cooperation, a series of achievement will be formed, such as data, formulation, report, software, photos, video, physical model and so on.

Finish the subject: Students show their achievement in the form of papers or reports online and communicate with each other, acquire others' remarks and suggestions to revise and improve their results. In the oral defense conference held in the real class, they can present their work .Teachers guide and give comments which consist of two parts. The first is the quality or accuracy of the mathematical model (the score for the quality).The other is comment which show the distinguish feature and creation, shine point of the work (the score for their creation).

The procedure for dual-class differs due to different content chosen in mathematical modeling. For example, it can be designed like this in function or mathematical modeling:

Learning objectives

1. To deepen understanding about the function by reading articles in the document folder on line.
2. Be able to find several real functions independently
3. Divide into groups, make their own task clear respectively,
4 . Learn how to comment and choose the best mathematical model.

Specific steps and learning requirements

Content and **credit hours**	Learning activities	Learning requirements and recommendations	homework
Real classroom (1 credit hour)	Introduce the mathematical modeling including its characteristics, requirements and function	In order to clear the direction and raise confidence, invite seniors to present their results and feeling.	
Learning Online (1 credit hours)	Students read materials about function	Be able to identify a variety of functions, to deepen understanding of them.	homework 1-1
Finding functions (0.5 credit hours)	Every student can find three real functions independently	Indicate where to find them and what functions they are.	homework 1-2
group sharing online and offline (1 credit hours)	1. Set up learning group. 2. Each group submits the results 3. Exchange and evaluate. 4. Self-evaluation.	1. Elect the leader from the group which includes the members and determine their respective responsibilities. 2. Select the best function this group has done, submit the process and outcome. 3. Select one or two other groups 'results and make their comments. 4. Submit the report, make a summary about the process and results and give the corresponding comment which should include: (1) Whether the whole process is complete, and whether the report for each step has been written. (2) The result is scientific so we can definitely find a function which we are very familiar with. (3) Show their creativity (finding a novel function , or a unique or special	homework 1-3 homework 1-4 homework 1-5

		relationship (4)If the comments on others' work are accurate.	
Identify the subject (0.5 credit hours)	decide the problem they want to solve using a function model	Among the problems provided by teachers ,Select one they want to solve or choose others after several discussion in group	

From the table above, we can see the whole process clearly which can be fixed gradually so that a stable teaching mode for function modeling can be formed.

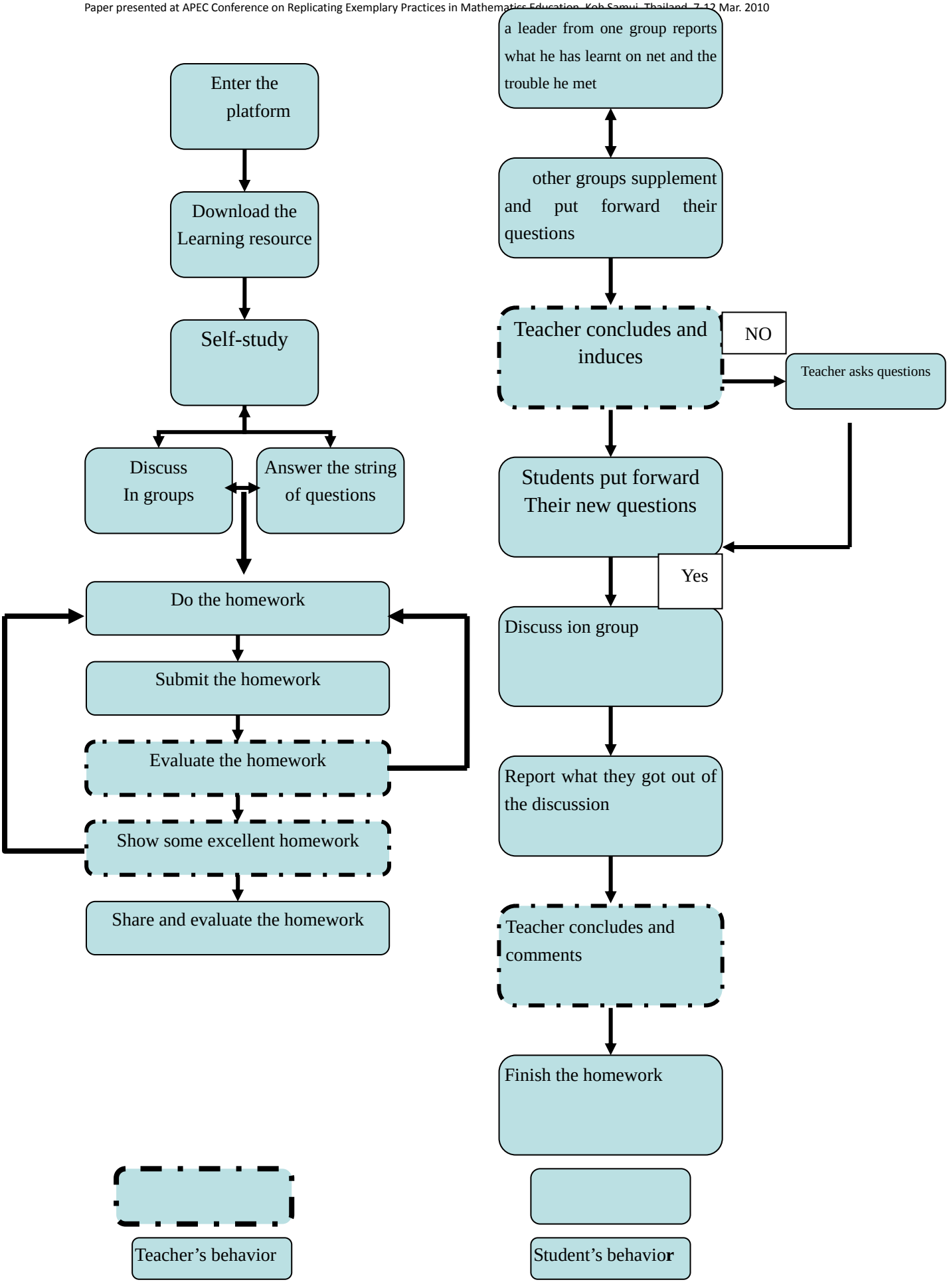
Case 2 "Double Class-Real and Virtual Class" teaching

The compulsory chapter "Solid Geometry Introduction" includes : The features of cylinder, cone, 台体, ball, combination, the drawing method of visual figure, stereogram and deepening the comprehension in the definition ,nature of parallelism or vertical as well as the critical theorem for such relationship between lines and surface in space. We divide them into six sections:

- Learn cuboids and common geometric objects.
- Cuboids and the drawing method of visual figure, stereogram 3.The position or relationship between lines and lines in a cuboid.
- The position or relationship between lines and surfaces in a cuboid.
- The position or relationship between surfaces and surfaces in a cuboid.
- Review cuboids again.

Each section is taught in both real class and virtual class, in which a PPT, a string of tasks, homework and relevant materials are necessary. We design the content in the form question string, which lead students to think by steering them in the right direction, and thus the desired objectives are easily attained.

（1）procedure for virtual class　　（2）procedure for real class


Enter the platform
Download the Learning resource
Self-study
Discuss In groups
Answer the string of questions
Do the homework
Submit the homework
Evaluate the homework
Show some excellent homework
Share and evaluate the homework
a leader from one group reports what he has learnt on net and the trouble he met
other groups supplement and put forward their questions
Teacher concludes and induces
NO
Teacher asks questions
Students put forward Their new questions
Yes
Discuss ion group
Report what they got out of the discussion
Teacher concludes and comments
Finish the homework
Teacher's behavior
Student's behavior

7. Summary and prospect

Teaching mode for ‘dual-class’ is still in the exploratory phase and some problems needs to be solved and improved. At present, the main challenges are:

1. What content in the text book can be taught in the dual-class?
2. How to design our new unit curriculum? How to arrange the credit hours? How to design the question string? What kinds of resources are proper?
3. Environment in the virtual class should be improved to make the exchange between students unobstructed.
4. How to input or compile the formula quickly and draw various geometric figures. How to connect mathematical software with network well.

The “Double Class-Real and Virtual Class” has brought great changes for both students and teachers ,break the time and space limit, motivated students’ passion and interests in math. Hence, we should believe this new teaching mode and endeavor to spread it. Here are some suggestions:

1. Setting up several schools to be experimented for further explore so that we can make sure what content can be involved in dual-class.
2. Setting up several unit curriculums for dual class to make the new mode easy to spread and resources easy to share.
3. Spreading this new teaching mode gradually to make its value accepted by more teachers and be used in their teaching.

Chinese Reference

『1』 Ministry of Education of the People's Republic of China (MOE).Ordinary High school mathematics curriculum standards (trial version) [M]. People's Education Press, 2003.

『2』 Ye Qixiao. Mathematical modeling teaching activities and Mathematics Education Reform [M]. Hunan Education Press, 2003.

『3』 Shangzhi Wang Unscramble of Mathematics Curriculum Standard of China [M] Jiansu Educational Press 2004

『4』 Shangzhi Wang,etc. Teaching Cases of China [M] Higher Education Press, 2005.

『5』 Zhang Siming Bai Yonhxiao. The practice and exploration of Mathematics learning [M]. Higher Education Press, 2003.

『6』 Development group for mathematics curriculum standards. Interpretation of Full-time compulsory education mathematics curriculum standards (trial version) [M]. Beijing Normal University Press, 2002.

『7』 Development group for mathematics curriculum standards. Interpretation of ordinary high school mathematics curriculum standards (experimental) [M]. Jiangsu Education Press, 2006

『8』 Wang Shangzhi,etc. A series of CD : "Mathematical Modeling Entering into the Classroom in High School" .Central Radio and TV University Audio Visual press 2006.09

『9』 Zhang Siming, etc. New high school mathematical curriculum and learning [M]. Higher Education Press 2008.08

『10』 Wang Shangzhi, etc. Enter into the new high school mathematics curriculum [M]. East China Normal University Press, 2007.09

『11』 Zhang Siming, etc. Teaching Design and practice cases of the project [M]. Beijing Normal University Press 2006.07

『12』 Zhang Siming. Stimulating creativity with mathematics. Mathematical papers 2006 12th edition

『13』 Ma Ping,etc. Let the subject learning become the creative stage. Mathematics journal of secondary school. 2008 1th edition

『14』 John Parker R.L.Moore: Mathematician and Teacher, Column Editor: Osmo

Pekonen Agora Center 40014

『15』 J. R. Kline to R. L. Moore, undated and incomplete letter, Box 20, R. L. Moore Archives, Center for American History, The University of Texas at Austin.

『16』

Zippin, Leo, A study of continuous curves and their relation to the Janiszewski-Mullikin theorem, *Transactions of the American Mathematical Society* **31** (1929), 144-162.

『17』

Zippin, Leo, On continuous curves and the Jordan curve theorem, *American Journal of Mathematics* **52** (1930), 331-350.

『18』

Moore, R. L., Sets of metrical hypotheses for geometry, *Transactions of the American Mathematical Society* **9** (1908), 487-512.

[19] Moore, R. L., A note concerning Veblen's axioms for geometry, *Transactions of the American Mathematical Society* **13** (1912), 74-76.

[20]Moore, R. L., On the foundations of plane analysis situs, *Transactions of the American Mathematical Society* **17** (1916), 131-164.

[20]Moore, R. L., On the foundations of plane analysis situs, *Proceedings of the National Academy of Science* **2** (1916), 270-272.

[21] Moore, R. L., A characterization of Jordan regions by properties having no reference to their boundaries, *Proceedings of the National Academy of Science* **4** (1918), 364-370.