LESSON STUDY AS A STRATEGY FOR CULTIVATING MATHEMATICAL TEACHING SKILLS A CHILEAN EXPERIENCE FOCUSED ON MATHEMATICAL THINKING

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Introduction

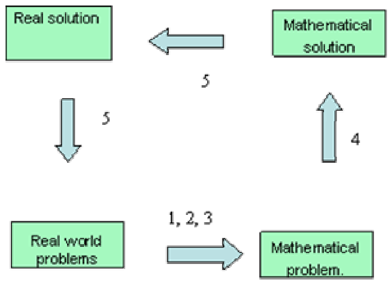
In Chile, approximately 100% of school age children are registered in primary school. Nevertheless, the school attendance does not ensure that mathematics learning is of the expected quality. The testing results indicate this lack of quality in math learning. There is great interest in our society as a whole to focus on a qualitatively change in the practice of math teachers. Chile has been carrying out multiple initiatives in this sense for several years and lately it is participating in a project to improve the education of mathematics with the technical assistance of Japan. At the same time, within the framework of the collaboration between the economies of APEC, Chile participates in a collaborative project in mathematical education that contemplates a progressive study of different subjects related to mathematical education: good practices, mathematical thinking, communication, evaluation, and generalization towards other subjects. This report gives an account of an experience that, following the methodology of Lesson Study, was developed according to the didactic principles of Consultant's School Strategy for the Curricular Implementation in Mathematics, strategy developed in Chile. The comparative analysis of the two previous strategies, made by Gálvez (2006), *allows us to conclude that both constitute powerful strategies to improve the teacher practice and, at the same time, to generate processes of professional learning of the teachers, which guarantees a greater stability in the changes obtained by its performance*.

Lesson Study focused in the mathematical thinking.

To focus the processes of study in the development of the mathematical thinking implies a double challenge; it involves the students as much as math teachers. Professor Katagiri (2004) aiming at the autonomy of the students indicates: "*Cultivating the power to think independently will be the most important goal in education from now on, and in the case of arithmetic and mathematical courses, mathematical thinking will be the most central ability required for independent thinking*".

On the other hand Stacey, K. (2006) emphasizes that "*providing opportunities for students to learn about mathematical thinking requires considerable mathematical thinking on the part of teachers*"

A key window considered in Chile to develop the mathematical thinking mentions the mathematization of real world phenomena. The cycle of mathematization has been described by means of the following scheme: (Jan de Lange, 2006):



1. Problem situated in reality
2. Identification of the relevant mathematics
3. Gradually trimming away the reality
4. Solving the mathematical problem
5. Interpret the meaning of the mathematical solution in terms of real world.

To connect a problem of the real world with mathematics is not trivial and requires a fundamental mathematical competence. According to (OECD-PISA), today at least eight key mathematical competences are distinguished, that allow this connection; they are: to think and to reason, to argue, to communicate, to model, to create and to solve problems, to represent, to use language and symbolic operations, formal and technical and, to use aids and tools.

The Consultant's School Strategy for the Curricular Implementation in Mathematics (LEM), sustained by the Ministry of Education in alliance with Universities of the country, is based on the following didactic principles for the learning of mathematics (Espinoza, 2004)

1. In order to learn, the student must take part significantly in the mathematical activity, and not only limit him/herself into accepting and applying the strategies taught or “shown” by the teacher.
2. Learning consists of a change of stable strategy, the replacement of knowledge by another one, caused by an adaptation to a situation.
3. The mathematical knowledge arises from the work of the children as the optimal answer to specific problematic situations that require it.
4. The learning activities must constitute true challenges for the children, when putting in crisis/conflicts their previous knowledge. These activities must be accessible to the children and have their frame of reference in familiar and significant contexts.
5. When the teacher (or the text) gives the necessary instructions to do the task correctly, it is the teacher who is using the required mathematical knowledge and not students.
6. The mathematical knowledge in a learning process must appear as the necessary knowledge to pass from the initial strategies, low efficient and inadequate, to the optimal one.
7. The students choose and share different resolution techniques. The “error” is a substantial part of the learning process.
8. The knowledge and mathematical procedures used must be valued by all the children. It is recommendable not to spend a long time between the moment at which the mathematical knowledge has emerged for the children, and the moment

at which the teacher emphasizes and systematizes it. But, it does not have to be formalized prematurely.

9. The students must have the opportunity to work and to deepen the knowledge until obtaining a significant dominion of it.
10. The argumentation and mathematical explanation are seen as laying the basis for the adjustment of the algorithms and the modification of the mistakes.

Description of a Lesson Study experience in Chile focused on Mathematical Thinking

This study took place in the Municipal School *Dr. Luis Calvo M.* (Santiago), but it had the participation of other teachers as observers, they belonged to two other schools (*R.I. School* and M, A.C. School), and they wished to become future developers of the designed lesson. The criterions to choose the school were, they have at least, one course per level, to have the conditions to develop an authentic experience, not to simulation, and the schools belong to Whole School Pilot Project (supported by the Ministry of Education and constituting part of Consultant's School Strategy for the Curricular Implementation in Mathematics). The process that is described in this work contemplated six phases that are described briefly next:

Phase 1 Presentation of Lesson Study Project to the group of invited teachers.

The first conversation was carried out in the school Luis Calvo M. and was attended by the mathematical teachers of that school plus the invited ones. They observed a PPT with the foundations of the Collaborative APEC Project in Chile, whose long term goal is to promote the development of mathematical thinking from the perspective of the mathematization processes. The author of this report assumed the role of external adviser, summoning the group and carrying out the presentation of the project. They discussed the following text (Takahashi, A., 2006): *the idea of lesson study is simple: collaboration with fellow teachers to plan, observe and reflect on lessons developing a lesson study, however is a more complex process Because lesson study is a cultural activity, an ideal way to learn about lesson study is to experience it as a research lesson participant. In so doing, you will learn such things as how a lesson plan for lesson study is different from a lesson plan that you are familiar with, why such detailed lesson plan is needed, what type of data experienced lesson study participants collect, and what issues are discussed during a post lesson discussion*.

One of the teachers stated that this group had elaborated, last year, a Didactic Unit (from now on DU) of Geometry in a course of teacher training and that had been only applied by one teacher, in October, 2006, but the other designers could not observe it. This DU was directed to 8^{th} grade; its subject was the area and perimeter of circumferences and consisted of at least eight classes. Since the subject of geometry is of particular interest, they agreed to start off with a critical revision of this DU before carrying out a new design. They agreed, in addition, to invite the teacher who had applied the Didactic Unit with the purpose of knowing first hand, how was this process. He was given a task of reading and of studying that Didactic Unit. Eight sessions were set up as a minimum to develop this project. All the assistants valued the opportunity to meet and reflect on the education of mathematics. They said that they had the idea to form a reflection group on mathematical education, that they would call *Pythagoras Group*.

Phase 2 Critical revisions of the subject area and perimeter of a circle

Teacher C.M, who worked on this subject, told them how he lived the experience when he applied the didactic unit designed by them. In particular, the discussion focused towards the second class that corresponds to the characterization of the number π. The class contemplated three moments:

- Measurement of the contour of circular base objects, using a ruler or measuring tape.
- Searching for a broad approach of how many times the diameter fits in the circumference (arriving at that it is 3 times and "something more". The lacking difference is not quantified.
- from measurements of the contour **P** of a circle and its diameter **d**, calculates the reason: **P/d**

The following photos illustrate the moment at which the students find that the diameter fits at least three times in the contour of the circumference. In doing this they bordered with plastilina the contour of the circumference.

There was an intense and interesting discussion that was centered in the following key points:

- Why the students did not quantify how 3 diameters need to cover the perimeter of the circumference? This course already has the tools to have done it (but, they only obtained value three).
- The plastilina had a problematic performance because it is tensile. Some children stretched it to fit it into the circle contour three times.
- When they did the quotient between the measures of the perimeter and diameter of the circumference (cm.), obtained numbers with three or more decimal, but those decimals cannot be considered as valid values; there exist a physical limitation in the measurement that prevents obtaining more precision than mm

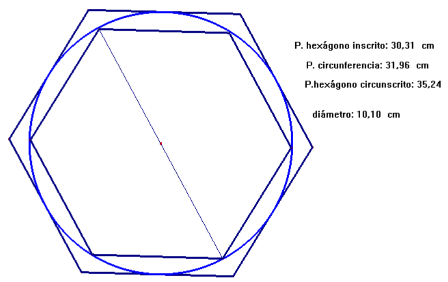
gives. In spite of this, the students obtained decimal values for the quotient, like the following ones.

	Perimeter cm	Diameter cm	Perimeter/diameter
Circle 1	25,1	8	3,1375
Circle 2	19	6	3,167
Circle 3	12,7	4	3,175

- The Teachers' team decided to focus their lesson design in the measuring of the contour of the circumference using as a unit of measurement its diameter and **quantifying** the magnitude of the remaining segment (smaller than a diameter).
- Another question that arose: which is the better moment for presenting the number π? Since it is very common in present education that π is defined early as the quotient **P/d.** They thought it was not necessary a premature definition of this number, it is better to obtain first an approximate experimental value and then to characterize it in a stricter form.
- One of the teachers was requested that he simulate a class about this same subject matter, but under the traditional way. He did it, and then an interesting discussion took place around the differences in the mathematical thinking that is put into play in each modality. The first proposal of a class was sketched and a task was given: advancing in the design of the class in individual form.

Phase 3. Design the research-class.

The design of the research-class begins. It will center in obtaining a quantitative relation between the perimeter of a circle and its diameter. For this purpose they will set out a sequence of measurements of the contour of a disc, varying in each case the measurement unit that is used: initially without measurement instruments, then using a graduated ruler, continuing with a metric measuring tape, to culminate with the use of the diameter as measurement unit. Product of the previous process they will obtain a good approach for the number π.



Teacher A.F. raises the idea to complement later the work later of this class by means of an activity with the geometric software Cabri Géométre II that will consist in drawing a circumference with a inscribed and circumscribed hexagon in it, getting the respective measurements of the perimeters with the Cabri tool for measure, and comparing those values with the experimental values obtained in the designed class. Also he raises the idea that the students analyze a documental about the interesting history of the number π. There remains a task: to write up a draft of the class plan.

Phase 4. To complete the lesson, establishing the necessary materials.

The intentions of this session are: to raise the hypothesis about what will happen in the planned class, to prepare good questions for the students, to anticipate possible ways of solving the problems, to raise and to assign different tasks about the observation of the participants. They agree that the students will work in small groups and will have discs on which they will carry out the measurements. Each professor brought different circular objects. They decided on covers of round plastic containers. The materials and the activities are proven. When carrying out the quantification of how many times the diameter d fits in the perimeter P of the circumference, they obtained an approximated value of $3\frac{1}{9}$, of the reason P/d. The used technique consisted of the segment that exceeds 3d was marked on the paper tape (of length d) and then this one was bent successively. They checked that the segment fitted 9 times. They defined a sequence of two classes: In the first class the contour of a disc will be quantified, but in the course of the lesson the conditions of that quantification will change in a progressive form: they measure first with any no conventional unit, then with a ruler, later with a metric tape and it will be culminated with a **measurement that uses the diameter** as the unit of measurement. One hopes that the students quantify the leftover segment after applying 3 times the diameter in the contour of the circular object. That quantification, according to the previous knowledge would have to be expressed through a fraction of the type $P = 3\frac{a}{b}$ times the diameter.

Phase 5 the lesson was taught

The two designed classes were applied in a consecutive form and both were observed by teacher R.L of the same school. In addition the lesson was recorded by the knowledgeable other. This report will be centered in the analysis of class 1 only.

Phase 6 Discussion after the lesson implementation.

The objective of this phase is to put in common the observed things, as well as the performance of the teacher and the answers of the students, to propose ideas to improve the class plan and to make decisions with respect to giving continuity to the joint work. The group of teachers evaluated class 1, through the observation of a video clip of 30 minutes. From that reflection there were the following conclusions:

- In general the class was developed with fluidity and they noticed that the students were involved in it.
- The closing of the class was missing with questions directed to the students, nevertheless the professor made many questions to the groups but not to the entire course.
- With respect to the lived experience, the teacher who led the class indicated: "*I could not always teach classes with this same modality. As an activity for introducing a subject, I agree, I believe that the richness of the learning comes from this, but gradually or at some time it is necessary to return to the role of the "demanding mathematical teacher", because they are going to face new texts, new institutions, other teachers and they have to be prepared*."

Another teacher responded to him: *"We are looking for the participation of the students in the construction of new learning. What happens in the traditional classes is to start off giving the value of π and the formula of the perimeter of the circumference. Here everything is oriented to the origin of π and to find out that the circumference perimeter is independent of the circumference size. If we understand that, we have made a significant advance in the understanding of everything that comes ahead."*

- About the role of the observer: he must have a more active role in the gathering of information. The observer teacher indicated that he felt a desire to interact in the class with the students by asking them some questions. In fact he did it more than once. It was recognized by the group that for a research-lesson like this one, it is very difficult for the teacher of the course to pay attention to everything that happens in the classroom.

In relation to mathematical aspects, they remarked:

- It is necessary to focus on having the students understand that there is a (liner) dependency between the perimeter of the circumference and its diameter. It was stated that there is not a good estimation of which is the measure of the contour of a circumference; in general, people think that it is less than the real measure. A teacher remembered that in certain professional activities they used a metric wheel that in each turn measured 1 meter, and asked the other teachers. Which is the radius of that wheel? And the answer took some time.
- In a another group, when measuring the contour with the diameter, one student said, "it fits 3 times and exceeds a small piece, but we cannot say that it fits 4 times". Another one said: it fits three times and 2 cm. (they mix the units: the diameter and the centimeter).
- With this work they arrived directly to an algebraic expression to find the perimeter of a circumference, it was the consequence of the previous process and it was necessary to formalize it. So far they had obtained a very good approach for π of $3\frac{1}{7}$
- The observer teacher indicated that the teacher missed insisting that in all cases it gives a constant value for the quotient P/d, independent of the size of the disc. *"There is not a little π for small circumferences, and a great π for large circumferences"*
- The teacher responsible for the class said: "I need the students to work with Cabri so that they see that π is a quotient comparison and it has the same value in any circumference", that is to say, π is a ratio. In relation to the question about whether these students have learned the concept of ratio, the answer is no, there is a need to discuss it further in order to see that this ratio is constant. The teacher concludes: he could review the subject about the meaning of π when the subject of proportions will be studied later in the class. It is necessary to arrive to other a problem in which π is used. Another teacher asked: In what other subject π is used? , and the external adviser indicated that in probabilities. There is an experiment of Buffon (century XVIII) that consists of throwing a needle of length

b, on a board divided by lines separated from each other by a certain distance ***a*** (smaller than ***b*** or just as ***a***). Knowing that the probability that the needle falls on one of the lines is 2b/ (a• π), so we can consider π like in the previous case. We can simplify the experiment taking ***a*** = ***b***, with which the probability of the event is 2 /π, and then dividing 2 by the frequency whereupon the event happens we will have our approach of π be the same result as when Buffon made the experiment throwing the needle and obtaining π until with 3 decimal numbers.

- When seeing the techniques that the students used to respond to the challenges raised in the class, we verified that there are some techniques that the teachers could not anticipate: "*When we thought about the class we did not imagine this.*" For example with the use of the ruler: that they were going to draw the circle on the paper and soon they would mark cords of 1 cm, one after the other. In the case of an 18 cm diameter circle, they managed to mark 56 cords of 1 cm, that is to say, they registered a regular polygon of 56 sides. The result has an error of less than 1%; we remembered that with a regular hexagon the error registered is of ≈ 4.5%.

Relative to the performance of the students

- In a group, when a student was asked to explain what he obtained, he refused by saying: "*I cannot express concepts with words*". Teachers reflect on this point asking themselves if it is a language problem. One of them indicated that the student did not know, therefore he cannot explain. Another teacher answer him that he does not think the same, since he observed that the student participated actively in the class, thus he thinks that he lacks mathematical language to express what was experimented. Another teacher commented that one of the key competences in mathematics is communication and that it is neglected in the mathematics class.
- The teacher who led the class indicated that his evaluation of the lesson was positive, he was impressed by the children movement in the room, the spontaneous discussion and the speech of each one of them: "I felt them to be true protagonist of their own learning."

ANALYSIS OF THE LESSON

The following analysis is taken from the Anthropological Theory of the Didactic. (Chevallard, 1997). The subject treated in this class can be considered like an isolated mathematical organization conformed by a mathematical task, the techniques that allow making this task and by a theoretical speech that allows us to explain the techniques and to give a theoretical sustenance to them.

Mathematical task of the class:

- To quantify the perimeter of a circle.

Didactic variables:

- Availability of a circular object (disc) in class 1 or the drawing of a circumference on the paper. (in class 2)
- the type of measuring instrument available to carry out the measurement, or the unit of measurement to be used in the quantification (example: the length of the diameter)

Conditions: these vary progressively throughout the class:

- does not have measurement instrument, only have the disc
- use of one ruler,
- use of a metric tape
- has only a measuring tape of equal length to the disc diameter.

Techniques:

- Uses pieces of paper like a unit of measurement
- Uses its fingers like unit of measurement
- Marks a point of the circular object and makes it roll on to the ruler until the marked point returns to the initial position.
- They copy the contour of the circumference in a sheet of paper and with a ruler they measure with cords of 1cm drawn up one after the other.
- Turn the ruler around the disc. They border the circular object with the paper metric tape measurer
- Put the tape on top of the diameter in the contour concluding that it fits 3 times, but it exceeds a segment smaller than the diameter.
- to quantify the segment that is left successively doubling by half the unit of measurement and obtain the fraction $\frac{1}{8}$ of the diameter as an approximate value (division of the unit)
- Cut a tape to the length of the leftover piece and successively put it on top of the diameter to find out how many times it is possible to fit it (repeat the measure of the piece as many times as it is necessary to cover the diameter completely).

In the development of the class three essential moments can be distinguished:

Beginning Moment: When the mathematical task of this class is presented and it consists of measuring the contour of a circular object. The students react at this moment, according to the conditions put forward by the teacher, without any conventional measures (they measure with the fingers or a piece of paper).

Development Moment: Starting from a progressive change of conditions (availability of ruler, tape, the diameter as unit of measurement) the students elaborate other techniques of resolution. Here the students are faced with diverse obstacles according to the instruments they use, if it is a ruler they have the problem of measuring a curved line with a straight instrument, a thing that is avoided when they use a metric tape, because this one can take the form of the circular object that is being measured. The culminating moment arrives when the students are asked to quantify the contour of the disc only using a piece of paper of equal length to the diameter of the disc. In the first approach, all obtain the result that the diameter fits three times in the contour, but exceeds a small arch that must be quantified when they only have a unit of measurement greater than the length of that arch. Thus, it is a problem, because normally one measures with units smaller than the object to be measured. This implies, necessarily, that the unit of measurement will have to be divided. It is here where diverse techniques arise to obtain the fraction of the diameter. Some students (successively) double the unit of measurement by half until they obtain the eighth, verifying that this fraction of the diameter is very close to the length of the leftover arch. Others mark, on the unit of measurement, the length of that arch and soon they double (successively) according to that measurement, to obtain, either a $\frac{1}{6}$, a $\frac{1}{7}$ or a $\frac{1}{9}$.

Closing Moment: the proposed tasks are reviewed and the techniques used are compared. The new knowledge is identified and institutionalized: the diameter of the circle (measured in cm.) multiplied by some of the following fractional numbers can obtained good approaches to the perimeter of a circumference by multiplying: $3\frac{1}{6}$, $3\ \frac{1}{7}$, $3\frac{1}{8}$, $3\frac{1}{9}$, $\frac{1}{7}$ being the best approach, since it gives the first two decimals in an exact same form: 3,14. The class culminates with a verification activity that consists of: each group must designate a member to go to the blackboard (adherent paper tape exactly to the length of the perimeter of their disc. Then he returns to his group and verifies that it is the right measurement to cover exactly the contour of the disc, without lacking or going over adherent tape. The group that does it well, wins. The class was called: "**How much does a wheel move in one turn?** The students learned to quantify the length of a complete turn of a wheel. From the point of view of modeling (one of the key mathematical competences), we can say that in this real world problem a mathematical structure was imposed: $P = 3\frac{1}{7} \bullet d = \pi \bullet d$, formula that relates the perimeter of the circumference with the length of the diameter.

Final Reflections, Conclusions and Projections

Having finalized this first part of the project we can indicate that it had many benefits for all the participants. The students were committed and enthusiastic with the work proposals; they enjoyed the activities, discussed among themselves and with the teacher the mathematical topic of the class. For the teachers it meant having a longed for space for dialogue and reflection about math education, not only with teachers of the host school, but that with teachers of two other schools. In relation to the institutional aspects, it is necessary to emphasize the strong support of the Director and the Technical Pedagogical Sub-Director of the School Dr Luis Calvo Mackenna, since they gave all the facilities to find the space and the time for meetings of the team that designed and tested this lesson study.. This reminds us of what a director of a North American school stated:

"If we are serious in the fight to constantly elevate the quality of the teachers we must provide to the teachers the time and the resources to them that they need to form a

School	Mathematical teachers who will implement the lesson	8th level courses	Students
Luis Calvo Mackenna	2	2	70
República de Israel	2	2	70
Miguel Ángel Cruchaga	2	3	120
Total	**6**	**7**	**260**

community that invests as much in the learning of the children as of the same teachers. If we are serious in not leaving no child behind, then we must provide a process so that the teachers assume the responsibility of a growth and continuous improvement that does not leave no teacher behind either". (Liptak, L. 2005)

Limitations

- Little time available for the teachers to meet, in order to plan and reflect. They have too many hours of class teaching in their contracts.
- Difficulties to carry out the observer role, for the same previous reasons. To observe the research-lesson means to be absent from their own classes.
- Necessity of a "knowledgeable other". At least during a period of time the intensive support of an external adviser person to the school is necessary. This support should probably go away smoothly until the participants gain experience and autonomy.

Projections

The host school is thinking to apply this research-lesson in another parallel class (8° B). The teachers of the schools that also participated as observers will apply this class in their respective classes; for this purpose a meeting will be held also with schools that did not participate in the project, but which are interested in applying it. The sustenance of this methodology has a good perspective in this school, since there is a supporting principal, depending on the disposition of the teachers to continue practicing it. The idea exists to invite teachers of the neighboring schools to participate in a session in which the class with real students is demonstrated and all the previous ones within the framework of a plan that can soon be discussed with them (LS Open House) to improve the learning of geometry. The following picture shows the quantitative projections of the application of this class:

Finally we wish to indicate that research demonstrates, (Nordenflycht, 2000), that the effectiveness of the changes in the educational practices is related to the level of participation of the involved ones in the process, of the degree of depth of the reflection and the analysis that they carry out in their own practice. On the other hand, the development of a common project requires the rupture of the isolation in which the teacher develops its professional work. This isolation in the professional exercise constitutes, without doubt, a brake for the generation of a collaborative work that is one of the foundations of professional development. Consequently, a proposal to improve has to make it possible for teachers to reflect on their own practice, a collaborative work in which the investigation and the innovation are closely bound in their role as guides and promoters of learning. It implies, in addition, to develop competences and strategies, to analyze and to interpret situations, and to promote viable and effective solutions and alternatives of qualitative improvement. An independent teacher is a subject able to carry out a design on his own, able to interpret his reality and its context, to take initiatives, in synthesis, a constructor of innovations.

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APPENDIX

General Information

Title: How much does a wheel move in one turn?

Topic: Perimeter of a circumference obtained from measuring the contour of a disc with a tape whose length is equal to the length of his diameter

Producer: Ministry of Education, CHILE.

Video recorder and video editor: Francisco Cerda Bonomo

Teacher: Alejandro Flores

Research Team: Rafael León, Alejandro Flores, Soledad Cortes, Christian Méndez

Knowledgeable Other: Francisco Cerda B

Collaborator: Ms. Grecia Gálvez

Host School:: Escuela Municipal Dr. Luis Calvo Mackenna. Santiago
Principal: Sr. Patricio Morales Borbal
Pedagogical Coordinator: Ms. Carmen Corvalán Fernández

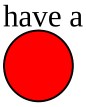
Invited Schools: Escuela Miguel Ángel Cruchaga, Puente Alto. Santiago
Escuela Municipal República de Israel, Santiago

Grade: 8th year of Primary School

Date: June, 2007

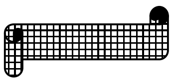
1. **LESSON PLAN**

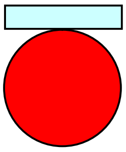
<table>
<tr><th>Lesson 1</th><th>Conditions
(teacher vary progressively throughout the class)</th><th>Techniques
used by the students who allow them to make the mathematical task under specific conditions</th><th>Remark</th></tr>
<tr><td rowspan="3">Mathematical task

to quantify the perimeter of a circumference</td><td>▪ They do not have measurement instrument, they only have a plastic disc
</td><td>• Use its fingers like unit of measurement
• Use a piece of paper like a unit of measurement</td><td rowspan="3">Ask to the students:

▪ what value you consider it will have the length of the contour? Write down it!

▪ What difficulties present the measurement with a ruler?

▪ What differences you consider exist between the measurement with a ruler and with a paper tape?</td></tr>
<tr><td>▪ use a graduated ruler (in cm and mm)
</td><td>• Marks a point of the circular object and makes it roll on to the ruler until the marked point returns to the initial position.
• They copy the contour of the circumference in a sheet of paper and with a ruler they measure with cords of 1cm drawn up one after the other.
• Turn the ruler around the disc.</td></tr>
<tr><td>▪ use of a metric tape
</td><td>• They surround the disc with a metric tape</td></tr>
</table>

	Using a piece of paper of equal length to the diameter of the disc. Students will be requested to quantify the magnitude of the segment that exceeds 3 times the diameter. (This remaining segment is smaller than a diameter). 	• Put on top of the diameter in the contour concluding that it fits 3 times, but exceeds a segment smaller than the diameter. • to quantify the lacking segment successively doubles by half the unit of measurement and obtains the fraction 1/8 of the diameter like approximate value (division of the unit) • Cut a tape of the length of the leftover piece and successively put on top it in the diameter to find out how many times it fits is possible (to repeat the measure so many times of the piece as many times it is necessary to cover the diameter completely).	What fraction of the diameter is the leftover segment? They will be asked for to cut a piece of tape equal to the contour length of the disc.

2. EXPLANATION OF VIDEO

❖ **Title of VTR**: *“HOW MUCH DOES A WHEEL MOVE IN ONE TURN”?*

❖ **Summary**

This video shows a lesson for the eighth course of primary school. The study subject talks about the quantification of the perimeter of a circumference from the measurement of the contour of a circular object. From the conditions that the teacher is putting for the measurement, a progressive elaboration of techniques on the part of the students takes place. The central activity consists of which they measure the perimeter of the circular object using a tape whose length is the measure of the diameter of that object. One hopes that the students quantify that measurement using the whole unit of measurement and a fraction of it

❖ **Components of the lesson and major events in the class**

In the development of the class three essential moments can be distinguished:

Beginning Moment: When the mathematical task of this class is presented and it consists of measuring the contour of a circular object. The students react at this moment, according to the conditions put forward by the teacher, without any conventional measures (they measure with the fingers or a piece of paper).

Development Moment: Starting from a progressive change of conditions (availability of ruler, tape, the diameter as unit of measurement) the students elaborate other techniques of resolution. Here the students are faced with diverse obstacles according to the instruments they use, if it is a ruler they have the problem of measuring a curved line with a straight instrument, a thing that is avoided when they use a metric tape, because this one can take the form of the circular object that is being measured. The culminating moment arrives when the students are asked to quantify the contour of the disc only using a piece of paper of equal length to the diameter of the disc. In the first approach, all obtain the result that the diameter fits three times in the contour, but exceeds a small arch that must be quantified when they only have a unit of measurement greater than the length of that arch. Thus, it is a problem, because normally one measures with units smaller than the object to be measured. This implies, necessarily, that the unit of measurement will have to be divided. It is here where diverse techniques arise to obtain the fraction of the diameter. Some students (successively) double the unit of measurement by half until they obtain the eighth, verifying that this fraction of the diameter is very close to the length of the leftover arch. Others mark, on the unit of measurement, the length of that arch and soon they double (successively) according to that measurement, to obtain, a $\frac{1}{6}$, a $\frac{1}{7}$ or a $\frac{1}{9}$.

Closing Moment: the proposed tasks are reviewed and the techniques used are compared. The new knowledge is identified and institutionalized: the diameter of the circle (measured in cm.) multiplied by some of the following fractional numbers can obtained good approaches to the perimeter of a circumference by multiplying: 31/6, 31/7, 31/8, 31/9, 31/7 being the best approach, since it gives the first two decimals in an exact same form: 3, 14. The class culminates with a verification activity that consists of: each group must designate a member to go to the blackboard carrying only a tape of equal length to the diameter of the disc of its group and he must cut an adherent paper tape exactly to the length of the perimeter of their disc. Then he returns to his group and verifies that it is the right measurement to cover exactly the contour of the disc, without lacking or going over adherent tape. The group that does it well, wins. The class was called: "How much does a wheel move in one turn? The students learned to quantify the length of a complete turn of a wheel. From the point of view of modeling (one of the key mathematical competences), we can say that in this real world problem, a mathematical structure was imposed on it: $P = 3\ 1/7 \bullet d = \pi \bullet d$, formula that relates the perimeter of the circumference with the length of the diameter.

- **Possible issues for discussion and reflections with teachers observing this lesson**

1. What may be the goals of this lesson?
2. How can we characterize the mathematics of this lesson?
3. How does the teacher view his students?
4. What are the characteristics of the classroom management of this teacher?
5. Is there more mathematics stakes in this problem of which the teacher should be aware?
6. What may be the learning outcomes and the follow-up for such lesson?