A STUDY OF "GOOD" MATHEMATICS TEACHING IN JAPAN

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The aim of this paper is to advance understanding on the characteristics of the lesson which is often recognised as a "good" lesson in Japan from two perspectives: learning process and teaching process. A case study will be carried out using the theory of didactical situations on a videotaped lesson which according to Japanese standards is a "good" one.

INTRODUCTION

What is "good practice" or a "good lesson"? The adjective "good" is subjective. We do not have an absolute criterion for "good". Also, what is "practice"? This term also indicates different activities. When the adjective "good" is used, the object has to at first be clarified. So that the participants may share the ideas on mathematics teaching activities of different countries, on the occasion of the previous APEC specialist session in Tokyo, a discussion was raised on these questions. It seems that the meaning of the later term, "practice", has been well clarified among the participants: it refers to the teaching practices in a classroom on the one hand and on the other the teachers' practices which allow their professional development and consequently the improvement of teaching practices in the classroom. The "Lesson Study"[1] well developed in Japan is thus often recognised as a "good practice" in this later sense[2].

However, the answer for the first question about "good" was not easy to discover. In the case of "good" teaching practices or lesson, some criteria based on different viewpoints were proposed, out of which here below three of them are summarised.

- Teaching process
  One way to define "good" is by the teaching method. A "good" lesson is given by adopting in the classroom a method recognised by the teachers as a "good" approach for teaching. For example, the lesson by the open-ended approach (cf. Becker & Shimada, 1997) is often recognised as a "good" lesson in Japan.
- Learning process
  The postulate of the constructivists, "Pupils construct their own knowledge, their own meaning" (Balacheff, 1990, p.258), supposes that if the learning in the classroom were conducted so that the pupils or students could construct knowledge and meaning by themselves, the lesson would be successfully carried out and the teaching practice used would be a "good" one. To evaluate a given class, in this case, students' learning process should be precisely analysed from the constructivist point of view.
- Students' achievement or outcome
  The other way to define "good" is by assessing students' results. The lesson is recognised as a "good" lesson, if students have achieved well in the mathematics assessment. For example, we may evaluate students' progress from the results of the national or regional assessment. The students' achievement should be rated by the goals of the curriculum or the lesson. Some goals can be assessed by a simple paper

test, but others cannot be. The later is usually recognisable in the learning process. Therefore, this criterion of "good" overlaps the second criterion.

These criteria are often used together to identify or discuss on the subject of a "good" practice or lesson preparation.

The aim of this paper is to advance understanding on the characteristics of the lesson which are often recognised in a "good" lesson in Japan from the first two points of view stated above: learning process and teaching process. A case study will be carried out on a videotaped lesson which according to Japanese standards is a "good" one.

Theoretical Framework

The global image of teaching in Japan compared with that of Germany and that of United States has been enunciated in TIMSS video studies: "In Japan, teachers appear to take a less active role, allowing their students to invent their own procedures for solving problems" (Stigler & Hiebert, 1999, p. 27). The motto for Japanese teaching has been called: "structured problem solving" (ibid., p. 27) while the Japanese lesson pattern has been characterized through comparison with patterns of other countries by a sequence of five activities (ibid., pp.79-80)[3]:

- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major points

These patterns describe the overall activities which are conducted in the classroom. In order to analyse more precisely the characteristics of "good" teaching practice in this paper, the theory of didactical situations (Brousseau, 1997) is adopted as a tool of analysis. It is not a teaching method nor an evaluation method of the teaching practice, but provides us with a model for the analysis of an effective classroom in order to understand what processes are taking place in terms of students' learning. At the same time this theory allows us to identify the relevant learning and teaching situations (didactical situations) with reference to the mathematical situations.

In this theory, the Piagetian postulate for the learning is adopted: "The student learns by adapting herself to a milieu which generates contradictions, difficulties and disequilibria" (ibid. p.30). In order to characterise different process of learning and teaching of the target mathematical knowledge, four situations – action, formulation, validation and institutionalisation – according to the stages of lesson are taken into account. And the notion of "devolution" is also an important element for the analysis; it's a process in which the teacher puts the student in an "adidactical situation" (ibid. p.30) where the student solves the problem on his own responsibility. The learning and teaching situations are modelled by the notion of "games". The student's games are to play "with the adidactical milieu which allow the specification of what the function of the knowledge is after and during the learning" (ibid. p.56), and the teacher's games are to organise student's games. A didactical situation is therefore expressed as the diagram (Figure 1).

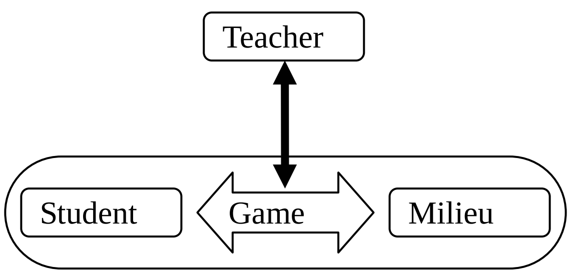


Figure 1: cf. Brousseau (1997, p.56)

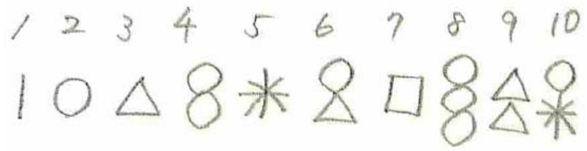
Theoretical Analysis of the Tasks

The lesson I selected is one which is given by a teacher of the elementary school attached to the University of Tsukuba. This was a part of a “lesson study” that is demonstrated on the occasion of APEC-Tsukuba conference in January 2006 in Tokyo. The teacher of attached school is recognised as a practised expert teacher.

Lesson topic: prime and composite numbers

The lesson plan written by the teacher is attached to the appendix of this paper. The target mathematical knowledge is the prime and composite numbers. The goal of the lesson is for pupils to be able *to view a number as a product of other numbers* (see the appendix). It will of course include the understanding of the fact that some numbers cannot be a product of other numbers except the identity element “1” and themselves. For example, the number 12 can be seen as a product of the numbers “3” and “4”. The lesson which will be analysed in this paper is the first of two consecutive lessons. The first one is the introductory lesson. We can find from the lesson plan the two main tasks proposed in the lesson:

Task 1: The cards are ordered. Identify the implicit “rules”.



Task 2: Using the discovered “rules”, how 11 and 12 can be expressed?

Each card has just symbols. The numbers in the above diagram are not given. The teacher’s expectation of task’s result is that pupils find the rules which allow them to accomplish the second task. Therefore they have to find and recognise that the circle corresponds to the number 2, the triangle to the number 3, the star to 5, etc. and some symbols together have a relationship of multiplication. If these activities are considered “games”, like in the theory of didactical situations, there are two games: the finding of implicit rules and the finding of the symbols for 11 and 12.

From the viewpoint of representation of the number, numerical representation and graphical or pictorial representation are taken into account. In the activities, alternation between numerical representation and graphical representation is proposed. The numbers expressed by numerical representation are not shown on the cards, but revealed in the teaching process. The advantage of the graphical representation is that it shows visually the structure of the number and the number system in terms of prime numbers, in other words, the composition by prime numbers. This point is very often concealed by the numerical representation.

Analysis of Task 1: implicit rules

For task 1, as the task for pupils is just to find implicit conventions, we may consider several rules. Some of them are operational for the second task and some of them are not. By clarifying the rules supposed to be found by the pupils, this analysis will help us grasp the nature of rules the teacher expected to be found by the pupils in an effective lesson. While analysing the rules, I made the distinction considering the nature of the statement, especially the validity of the statement, between the "descriptive rule" and the "hypothetical rule".

The former is a descriptive statement which is true in a given order of cards. It can be operational when it is applied to the cards whose symbols are unknown. For example, the following are descriptive rules which can be considered a priori, however their list is not exhaustive:

1. Some cards have only one symbol, whereas others have plural symbols.
2. If the number becomes bigger, the number of symbols increases as well, with some exceptions.
3. There is at least one circle in every two cards.
4. The even number has at least one circle.
5. There is at least one triangle in every three cards.
6. The numbers multiplied by 3 has at least one triangle.
7. The prime number has always one symbol, not more, and different from others.
8. The composite numbers have always more than two symbols.

The later rule is a general statement which is hypothetical but whose validity can be checked empirically in the given cards. For example, the followings are those ones:

9. A symbol represents a number.
10. A symbol represents a prime number.
11. A circle represents the number "2".
12. A triangle represents the number "3".
13. The composition of symbols represents the multiplication of numbers.

This distinction will be important because the descriptive rules can be found or stated directly from the observation of given cards and numbers, whereas the hypothetical rules need to be verified by using other hypothetical rules. Moreover, it's the descriptive rules that allow pupils to formulate hypothetical rules and these are the kind of rules expected by the teacher to be found.

The rules stated above are "correct" from the viewpoint of the teacher's expectation. We may also consider "false" rules. For example, the composition of symbols represents the addition of numbers. I want to also mention the relationship between the rules in the sense that there is a hierarchy among rules. In particular, in order to find and apply rule 13, some other rules, such as the rules 9, 11 and 12, should have been discovered and applied. This means that whereas the question posed for task 1 is open, some specific rules are required for task 2.

Analysis of Task 2

The second task is the "game" to find the graphical representation of given numbers and consists of two sub-tasks. The first one of these is to find the symbol for the number "11" and the second is to do the same for number "12". To accomplish the first sub-task, the discovery of the descriptive rule number 7 from the above list will make pupils anticipate the symbol on the 11th card as a single symbol. For the elucidation of the second sub-task several rules should be employed. A brief analysis of the nature of this second sub-task is included in the paper for clarification purposes.

We may consider as an approach for the resolution of the second sub-task the factorisation of given numbers. The process of resolution is as follows:

1. factorise at first the given number "12" in the numerical representation (e.g. 12 = 2 x 6);
2. find the symbols which correspond to the numerical numbers obtained by the decomposition (e.g. "2" to "o", "6" to "Δo", etc.);
3. draw them together one below the other.

In the first step, for the factorisation, division and multiplication are available. In the case of multiplication, pupil will find heuristically two or three numbers, multiply them, and verify whether their multiplications will be "12". The pupils who cannot decompose are not able to reach the answer. What indicates to the pupils they should factorise the given number are primarily the rules 8 and 13. In fact, if the composition of symbols is not recognised as number multiplication, the factorisation cannot be done. Hence, this step requires to implicitly or explicitly use rule 13 to accomplish the task. The second step consists of discovering the correspondence between the numerical numbers and the symbols. The hypothetical rules 11 and 12 are required. The third step will be solved by using again rule 13.

The hypothetical rules are required to accomplish the second sub-task. The descriptive rules are not enough, and such this latter set of rules allows pupils to anticipate the symbols on the 12th card, but not the complete suite. That is, rule 3 or 4 makes them anticipate that at least one circle will be on the card; rule 5 or 6 to expect at least one triangle on the card. However, these descriptive rules are not able to make them anticipate two circles.

Analysis of the Videotaped Lesson

The lesson is videotaped and analysed using the video recording and the transcript. The actual lesson was proceeding including several activities. The analysis here will be conducted by dividing the lesson into three stages: introductory activities, activities for task 1, and activities for task 2. Each part is described and at the same time analysed. This analysis is not a part of the lesson study, but is carried out on the videotaped data from a researcher's and not a teacher's point of view in order to clarify the characteristics of a "good" lesson in Japan.

Introductory activities

The pupils' activities at this stage were limited to replying questions or fulfil requirements which are chronologically described hereafter.

1. Pupils were asked what they have noticed on the cards introduced into the lesson by sticking them randomly on the blackboard. Among them two cards have no symbol;
2. Pupils had to come up with symbols that might be on the two blank cards;
3. They had to make suggestions of ways to categorise the cards in order to make clear some implicit rules (not necessary the ones they were supposed to find).

In the lesson, as there are not many criteria on which the answer to the second question can be based on, when a pupil proposes a grouping, the teacher asks whether there are or not any criteria for the classification of the cards. This question implicitly makes the pupils group the cards. The categorisation methods proposed by the pupils are generally summarised in the following two methods:

- By the number of symbols on each card
- By the combination of different symbols on each card

At this stage, we see that pupils are familiar with symbols and recognise or find the implicit descriptive rule of the distinction between one symbol and composed symbols. It also seems that the pupils are aware of the differences and similarities of the type of symbols in the given cards and symbols (see the following dialog).

22. S: The card with a bar should go to the second group because it has only one symbol.
23. T: So there are groups of one symbol, two symbols and three symbols. *This is a pattern easy to see for everyone!*
24. S: All the cards of the first group *have* a circle, so the card with two triangles and no circle should not be there.
25. T: *So you are saying* that these are all circles so the triangle should go to the other group. This can be one way of thinking.
26. S: *The cards with two different symbols belong to the first group, but the card with two circles should go the bottom group where there are cards with only the same kind of symbols.*

From the viewpoint of the theory of didactical situations, this stage is the first step of a devolution process which allows pupils to better understand the rule of the "game" that the teacher will propose later (find implicit rules in task 1 and find symbols for given numbers in task 2). No matter what, the teacher does not directly ask pupils for the next activity, but uses pupils' discourse and guides them (e.g. "so you are saying …" [25]). This way of intervention on the part of the teacher makes pupils engage in their activities out of their own responsibility. The teacher's authority does not dictate their intellectual activities. This is one of the conditions for the devolution process.

Activities for Task 1

Task 1 is proposed to the pupils [29]. The teacher sticks the cards slowly one by one on the blackboard from left to right. He implicitly indicates the order of cards. At this moment, the cards have not yet been given numerical figures. As the teacher is sticking cards slowly, the pupils anticipate which should be the next card to be put up. At this moment, the goal of the "game" for the pupils is to find implicit rules ("please tell us what kind of pattern you found" [29]) and at the same time to find the next card.

29. T: […] Now, I'm going to reorder these cards in my way. By looking at my way of ordering, *please tell us what kind of pattern you found*. The way of thinking you did will be very helpful. I'm putting the first card, the second one, the third…

Just after sticking all the cards on the blackboard, some ideas are proposed by the pupils. One of them is "S: it's a multiplication" [36]. Although this is the final rule the teacher expects to be discovered, he writes it down on the blackboard and asks to the class to find simpler descriptive rules: "T: the hint does not have to be so complicated. Can you see some more interesting rules in this pattern?" [43]. It is visible from this example that the teacher regulates the class on the path he was expecting it to go and the rules he is expecting the pupils to find at first are descriptive ones. The answers given by the pupils to the teacher's question are as follows.

- the even numbers have circles on the card
- the triangle appears after every two cards

These are the descriptive rules. As the numerical numbers have not yet been written on the cards, the teacher asks the pupil who proposed the first descriptive rule to write down numbers in order to clarify his proposition for the other pupils. The teacher clarifies the pupil's idea. At this moment, the rules found are not the final hypothetical ones, but the descriptive ones, which will play an important role in finding the final rule. Because of this, the teacher accentuates this rule for the pupils in the class: "T: It's an interesting discovery! Each card positioned on an even number has a circle" [51].

Until this point, the implicit rules proposed are focused especially on the relationship between the cards and the descriptive rules have been identified. Next, one pupil mentions the relationship between the symbols as it follows, and the teacher continues from there:

56. S: The number "2" has one circle and "4" has two circles, so I thought… two and two makes four. And next is "6"…
57. T: […] The circle means two. Two and two is four. O.K? Do you understand? Two and two is four. *How else can you say that in mathematics?*

The first pupil's proposition "two and two makes four" contains an ambiguity and also it seems that the pupil is not so certain of his answer [56]. The teacher at this moment clarifies the pupil's discourse, and makes pupils focus on it by saying "How else can you say that in mathematics?" [57]. This question makes pupils to think about the relationship between composed symbols and formulate the idea. In the words of the theory of didactical situations, the teacher puts pupils into a situation of formulation. Until this moment, pupils act and reflect on the given task in order to find the implicit rules in the given ordered cards and symbols. Therefore, they were in the situation of action. However, the distinction between action and formulation is not obvious, because the problem is to find a formulated rule.

58. S: 2 plus 2 equals 4. 2 times 2 equals 4.
59. T: both of them are right, no? … These four people seem to say no. So, please explain why you are against.
60. S: I think it is correct that the two circles of the 4th card mean the addition of two "2"s or multiplication of two "2"s, but if so, when it comes to the 6th card, *the triangle should represent number "3"* and then we got "3" plus "2", which is five. So, I don't think it should be an addition.

Next, the teacher proposes the validation of given rules [59], after multiplication and addition are both proposed [58] (situation of validation). Some pupils explain that the implicit rule which they are searching is multiplication rather than addition by using the other numbers [60]. After the pupils' explanations, the teacher summarises and verifies this rule for the other numbers on the blackboard: 2 x 2 x 2 = 8, 2 x 5 = 10. Then he states and writes down clearly on the blackboard that the implicit rule which they are searching for is "multiplication" (situation of institutionalisation).

The feedback from the milieu, when a pupil anticipates an implicit rule, will be the result of verification with other cards. For example, when the hypothetical rule "the composition of symbols represents addition" is anticipated, what validates it is the calculation result for 6 or 8 based on the other hypothetical rules. This feedback will be elicited from the milieu on condition that the pupils are aware of the rule validation method. In fact, if a pupil thinks that the rule to be found can be valid not for all but only some cards, this validation method will not be adopted. In the activities of this stage, the feedback from milieu would be situated in the situation of validation rather than in the situation of action.

Activities for Task 2

After his summarizing what has been found on the hidden rule, the teacher asks the first sub-task of task 2: what symbols will be on the 11th card? While asking the pupils, he also inquires whether the symbols which were drawn before (three triangles "ΔΔΔ") can be used for the 11th card. The answer from pupils comes out quickly: "27" expressed by three triangles is not relevant, and a new symbol is necessary for the number "11". The teacher then asks to the pupils the reason of the answer [83]. At which a pupil replies as it is expected by the teacher [84].

(pupils draw a pentagon and an X)

83. T: This is correct. This is also correct. Why do you think these are correct? Are these symbols that you drew the right ones? *Why do you think it is O.K. to use symbols like these?*
84. S: The numbers which have only one figure, take "2" or "3" for example, can be divided only by "1" or the number itself. "11" as well can only be divided by one or itself, so the card has to have only one symbol like before with the triangle or rectangle.
85. T: You are talking about the opposite of multiplication, don't you? If you think about something using a combination of multiplications, we can't express the number "11" in that way. It can be expressed by an addition, but none of these cards represent additions. Right?

The activity for the first sub-task does not take long. The essential fact that some numbers can be only divided by "1" is clearly stated by a pupil [84]. Therefore one of goals of this lesson, which is understanding of the prime numbers concept, has been reached. The new idea is repeated and clarified with examples of multiplication by the teacher [85]. The term "prime number" was not verbalised by the teacher in the lesson, due to Japan's national course of study where it is introduced in the 9th grade.

From the viewpoint of the theory of didactical situations, in order to accomplish the given task (find symbols for "11"), the target mathematical idea in the lesson (some numbers cannot be a product of other numbers except "1" and themselves) is elicited. What makes pupils to come up with this idea is the first sub-task. In particular, the teacher's question on the reason of the selected symbol formulates this idea to the pupil himself (formulation) [83]. We can also find a sign that the class is about to discover this idea appears in the remark of a pupil from the previous stage [71].

71. S: I understood it's a multiplication, but how can we come up with the number "11" by multiplication?

The second sub-task is proposed by the teacher, finding what symbols will be on the 12th card? This time, he invites them to write down on their notebook. The teacher asks a pupil and he gives a wrong answer, six circles "oooooo". This answer is corrected by others who indicate that the given symbols are wrong, at which point the pupil writes again: four triangles "ΔΔΔΔ". The problem is that he uses the unwanted rule of addition, instead of multiplication and the teacher uses this opportunity to clarify what implicit rule are they after. Before indicating that these given answers are wrong, the teacher asks a pupil to write on the blackboard answers differing from them. A girl writes two circles and a triangle (ooΔ). The teacher asks her the reason. She explains it by "4 x 3 = 12" (oo and Δ).

102. S: (the pupil draws two circles and a triangle) Because so far, as 2 times 4 is 8, there are three circles, and as 2 times 3 is 6, a circle and a triangle are combined, as 4 times 3 is 12 and 6 times 2 is 12, it will be like that (each time, she refers to the symbols of previous cards such as 2 and 4 to 8, 2 and 3 to 6, 4 and 3, and 6 and 2).
103. T: These circles make "4", and four times "3" is "12". Great! Why did you think these symbols are wrong? (pointing to the four triangles)

104. S: in the case of 8, we multiplied 2 to 2 and 2, therefore also in the case of triangle, 3 times 3 equals to 9, 9 times 3 equals to 27, 27 times 3 becomes 81.

After clarifying the girl's explanation to the class, the teacher comes back to the previous answer (ΔΔΔΔ) and asks to the whole class why it is not correct [103]. A pupil explains why [104] and the teacher clarifies her explanation. However, when the teacher asks again for the other answers, a pupil draws two hexagons on the blackboard. This pupil does not give an explanation, but the reason will be elucidated by the other pupils [112, 113]. This pupil did not use the expected rule, multiplication, but an unexpected rule, addition again.

112. S: Perhaps, as there are two symbols for "6", I thought she joins the two symbols and makes a new symbol, the hexagon. So, she thought two hexagons express twelve.

113. S: perhaps, she made a new symbol, hexagon, for "6", because of 6. And as there are two hexagons and as 6 times 2 makes 12, so I think she made two hexagons.

After getting other answers (Δoo and oΔo) from the pupils, the teacher finished the lesson by saying "T: some people may still have difficulties in understanding the multiplication" and asking for bigger numbers to be represented, such as "100".

At this stage, I found in this class that the hypothetical rule "the composition of symbols represents the multiplication of correspondent numbers" expected by the teacher to be used in task 1 was sometimes not employed by some pupils. The answers with four triangles or two hexagons appeared from this reason. It means that there was no feedback of milieu for the answers given by these pupils. No use of this rule is directly related to the absence of the verification method which allows a feedback of milieu. Insofar the expected rule is used, the feedback will be given from by milieu. I found here the importance in the organisation of milieu by the teacher. However, I have to also mention the way of regulation or intervention in this lesson when the absence of feedback from some pupils is found by the teacher. It is not the teacher's direct intervention that gives feedback to the pupils. He only clarifies pupils' ideas and asks other pupils in the class whether they are correct or not and why. For example, the pupil's discourse [104] could be a feedback for the pupil who gave the wrong answer (ΔΔΔΔ). It's therefore the social interaction which allows feedback. This social interaction would not establish itself without the intervention of the teacher. The feedback was not given by the milieu itself but came from the social interaction enabled by the teacher. This is a negotiation of "didactical contract" trying not to owe all responsibility of validation to the teacher.

Concerning the targeted mathematical idea in this lesson (to view a number as a product of other numbers), as far as the anticipated rules – multiplication, correspondences between numbers and symbols – are employed, the pupils consider and use implicitly or explicitly this idea in order to find the symbols for the number "12". In particular, as the compositions of numbers' graphical representation explicitly shows, the pupils are aware of the idea clearly. In the last part of the activities for task 2, we can see that by asking the other way of expressing "12", the teacher tries to elicit an idea that the order of numbers in the product does not matter.

Discussion and Conclusion

What I analysed in this paper is only one case of many lessons given by a Japanese expert teacher. And this lesson was selected, due to the fact that some of the participants for the previous APEC specialist session in Tokyo will also be at this conference in Khon Kaen in Thailand. Therefore we cannot generate and conclude the results of analysis for all Japan. It is also true that some approaches well known in Japan – problem-solving oriented lesson, problem-discovery oriented lesson, etc. – might be more conform to the teaching and learning process indicated in the theory of didactical situations (see for example, *Japanese lesson study in mathematics at a glance* edited by Isoda et al.).

Let us come back to the initial question: *what are the characteristics of the "good" lesson or teaching in Japan?* As Stigler & Hiebert (1999) describe the Japanese lesson in the videotaped studies, "structured problem solving", the lesson analysed was organised so that the quite demanding problems are posed and the students invent their procedures or solutions. The teacher carefully designed and orchestrated the lesson. It seems that these aspects are recognised "good" part of Japanese mathematics teaching. Using the theory of didactical situations, we can explain them summarising in the following two points: the way of intervention of the teacher for the organisation or regulation of a class and the problem elaborated for the lesson.

For the first point, as we see in the analysis, the teacher quite rarely gives an answer or solution to the given task and he does not directly validate pupils' answer. He only asks the reason of a given answer (formulation), clarifies pupils' statements, and brings them to a common solution by respecting their ideas. Even though the pupil's milieu is not organised well enough to give feedback, it's not the teacher who gives feedback, but feedback from the other pupils is promoted by the teacher through social interactions. These actions, all have as a goal making a relevant didactical contract between the teacher and the pupils over mathematical knowledge.

For the second point, the teaching material elaborated for this lesson, in order for the target mathematical ideas (number as a product of other numbers and exception for some numbers) in the lesson to emerge. The graphical representation which allows to visualise the structure of numbers and number system is adopted. Furthermore, the problem, especially task 2, is set up so as to require these ideas as the means of establishing the optimal strategy to solve the problem or reach the goal of the "game". However, we have to also pay attention that this kind of teaching aid sometimes elicits a phenomenon called "metacognitive shift" in which the teaching aid becomes an object itself (Brousseau, 1997, pp.26-27).

As this lesson analysed in this paper was a part of lesson study which also demands criticism, this paper will end with a personal opinion taking into account the results of the analysis. As many factors are correlated to make one lesson, I could not propose a solution but mention just two points. First, it seems that the organisation of milieu for the pupils could be improved. In this lesson, even though the rule of multiplication plays a crucial role, multiplication itself does not receive a special status for some pupils more than addition. Due to this fact, some pupils use addition. As addition is more natural for people than multiplication for composed symbols (see the number systems developed in the world), it is necessary to elaborate a situation which allows pupils to give a special status to multiplication[4]. It seems that the way of questioning for task 1 was not clear enough to make some pupils find a hidden rule which govern the ordered cards and their continuation. Second, it seems that several situations, such as those of action, formulation, and validation were overlapped so much and could not take enough time to each situation.

Notes

[1] Lesson study is an approach of self training by in-service teachers for the improvement of teaching. It's very often practised in Japan. See for example, Stigler & Hiebert (1999).

[2] See the proceedings of APEC conference in Tokyo, for example, the paper presented by Inprasitha et al. (2006).

[3] It seems in the eyes of Japanese that the lesson analysed in the TIMSS video studies is rather "good" one.

[4] This is the opinion which participants in the discussion moment of this lesson also expressed.

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Mathematics Public Lesson Grade 4 Mathematics Instruction Plan

Teacher: Tsubota, Kozo, Vice-Principal, Tsukuba Fuzoku Elementary School

Research Theme: Examining instruction that focuses on “viewing a number in relationship to other numbers, such as a product of other numbers.”

1. Title Prime and composite numbers

2. About research theme

(1) Fostering rich sense of numbers

The current (2000) revision of the National Course of Study (2000) stresses that the goal of “fostering rich sense of numbers, quantities and geometric figures” is to be considered carefully. Since multiplication is introduced in Grade 2, a specific goal, “to view numbers as products of other numbers,” has been included. However, this is only one specific instance of developing “number sense” that must be addressed all the way though upper elementary school. Therefore, we must constantly address number sense intentionally. Today’s lesson proposes the treatment of numbers sense using the topic of “prime and composite numbers.”

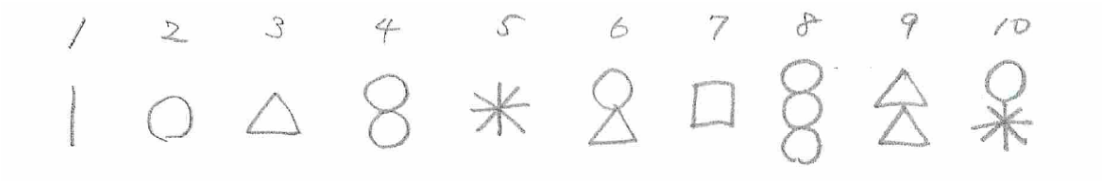
On p. 75 of Commentary on the Elementary School Mathematics Course of Study, you see a statement, “the goal is to develop an understanding of the multiplicative structure of numbers through an activity of counting objects by grouping.” Within the context of the introductory treatment of multiplication in Grade 2, this statement means that students should understand that a number can be viewed as a product of other numbers. For example, 12 can be thought of as 2 × 6 or 3 × 4.

In today’s lesson, we would like to further this perspective so that students can consider, for example,
12 as 2 × 3 × 3.

(2) Prime and composite numbers

In this lesson, we will pictorially represent the fact that all whole numbers are either prime numbers or composite numbers, which are products of prime numbers.

The following designs will be shown, and students are expected to identify rules the govern them. Then, using those rules, students will be developing designs for larger numbers.



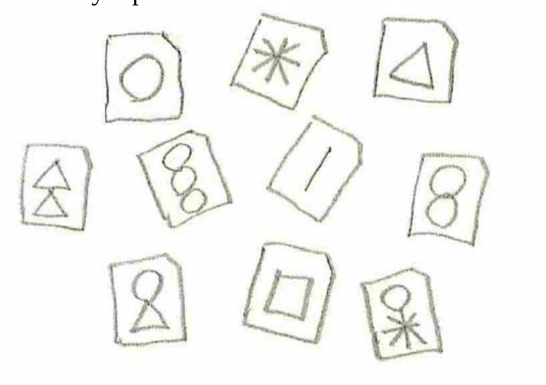
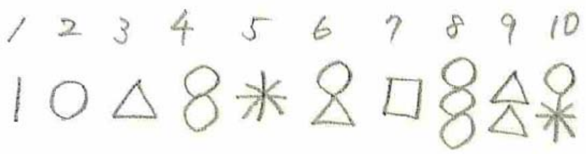
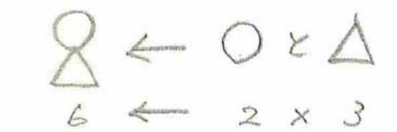
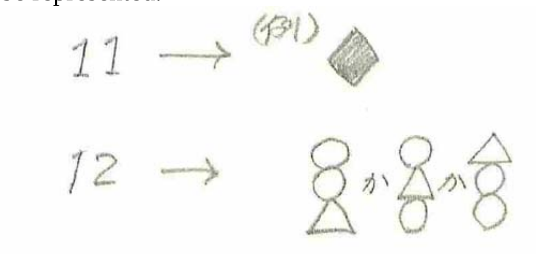
If students truly understand the ideas behind this lesson, they are more likely to understand the meanings of “least common multiples” and “greatest common divisors” to be studied in Grade 6.

3. Goals
To be able to view a number as a product of other numbers.

4. Instruction plan (2 lessons total)
   Understanding prime and composite numbers …… 1 lesson (this lesson)
   prime and composite numbers up to 100 ………… 1 lesson

5. Instruction of the lesson
   (1) Goals
   To notice that whole numbers are made up of prime numbers and their products.

   (2) Flow of the lesson

<table>
<tr><th>Instructional Activity</th><th>Points of Considerations</th></tr>
<tr><td>1. Observe the ten designs shown on cards and determine what they represent.
</td><td>(1) Post the ten cards on the blackboard at random. Ask students what they notice.
● If an idea that relates to numbers is raised, ask for the reasons.</td></tr>
<tr><td>2. Order the cards and identify "rules."

</td><td>(2) Guide the students to look at how the 6th design is composed.

</td></tr>
<tr><td>3. Using the discovered "rules," think how 11 and 12 can be represented.

</td><td>(3) Confirm that these designs represent numbers, then have them think about other numbers.
● Discuss and check the ideas for 11 and 12.
● Confirm that 11 must be represented by a new design while 12 can be represented by combining 2, 2, and 3.</td></tr>
<tr><td>4. Make a chart of number designs up to 20.</td><td>(4) Using the pattern they discovered, have students make the designs up to 20.</td></tr>
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