USING LESSON STUDY AS A MEANS TO INNOVATION FOR TEACHING AND LEARNING MATHEMATICS IN VIETNAM RESEARCH LESSON ON THE PROPERTY OF THE THREE MEDIANS IN A TRIANGLE

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Introduction

In Vietnam, the reform mathematics curriculum requires more than mastery of basic mathematical skills, good algorithms in solving a class of specific problems. The teaching of mathematics is changing. We are seeking for the innovation of teaching and learning mathematics. The teacher ought to think of teaching in terms of several principal hands-on activities, problematic real life situations, and open-ended questions. The innovation of teaching is to help students construct their own knowledge in an active way; enhance their thinking through solving non-routine problems while working cooperatively with classmates so that their talents and competencies are developed. There are several possibilities for innovation of mathematics education in an economy. Lesson study which originated from Japan is currently a central focus in US and other economies for the professional development of teachers and the improvement of students' learning.

In this research paper on lesson study for developing good lesson, we adopted a lesson study cycle comprising planning implementing and observing discussing and reflecting in our economy. The research focuses on lesson study as a means to innovation. The results from this lesson study showed that good teaching practices are powerful models for changing the quality of mathematics education. We developed a VTR of good lesson as a product of our lesson study and to use it for teacher education.

1. Planning

Since the Vietnamese secondary mathematics teachers who involved with this research were not familiar with the use of lesson study to improve their good practices in their classrooms. So first we had to conduct a workshop on "Lesson study as a means to innovation of teaching and learning mathematics". Twelve teachers, one specialist in mathematics attended this workshop; they were from the lower secondary school Nguyen Tri Phuong, Hue City, Vietnam. The objectives of the workshop were:

- to help teachers on how to use lesson study as a means to innovation of teaching and learning mathematics;
- to help teachers on how to use the innovation to improve students learning;
- to discuss with teachers on how the lesson study support the professional development of teachers;
- to help teachers on how to use innovation in teaching and learning mathematics to implement the reform mathematics curriculum;
- to select a well known and experienced teacher to prepare the lesson plan and carry out it in the class for observing and discussing.

At the end of the workshop, a group of teachers was formed to be involved in this study. This first team worked as a research group to create the lesson plan, worksheets and instructional materials suitable to the unit selected from the reform mathematics curriculum for grade 7 in Vietnam.

The unit "*The property of the three medians in a triangle*" chosen by teachers located in the text book, page 65-66, Volume 2, 2003. The teachers agreed in the meeting that the content of this unit is difficult and abstract to the students. The presentation of this unit in the text book is not meaningful. Traditionally, the students have to accept the definition of the median from the text book. The definition: "*The segment AM joining the vertex A of the triangle ABC with the midpoint M of BC is called the median of the triangle ABC*" is stated directly. In this research, teachers created problematic situations to help students explore the concept of medians and their properties meaningfully.

The study aimed to explore and investigate the implementation of lesson study as a means to innovation of teaching and learning of selected topics in lower secondary mathematics in Vietnam.

The research sought to find answers to the following questions:

1. How does the lesson study as a means to innovation affect to teaching and learning mathematics?
2. How does the innovation affect to the improvement of students learning?
3. How does the lesson study support the professional development of teachers?
4. How does the use of innovation in teaching and learning mathematics affect to the implementation of reform curriculum?

Findings of the study will shed light on the relative contribution of the lesson study as a means to innovation of teaching and learning mathematics.

The study was conducted in two months March - April 2006. All teachers were introduced to lesson study for the first time at the workshop of the research. Also at the workshop the methodology of the research was explained and discussed, i.e. that the teachers were responsible for their own use of innovation in teaching mathematics. What were required of them were observations on the things which happened in their classes and their reactions to the innovation.

Three classes were involved in the study. The students' ages ranged from 12-13 years. Overall a total of 145 students and 8 teachers were involved in the study. The study involved grade 7 students. The topic covered in the grade 7 was the property of the three medians in a triangle.

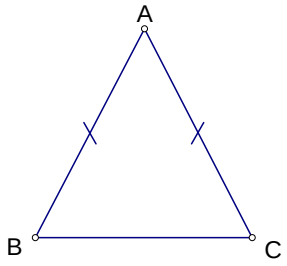
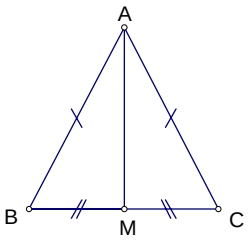
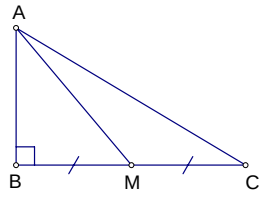
To prepare the lesson plan, we considered the role of this unit in the curriculum and discussed what teachers usually taught this unit. Teachers agreed that the lesson plan should have some characteristics as follows:

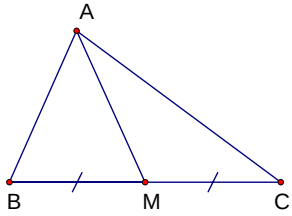
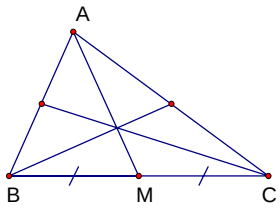
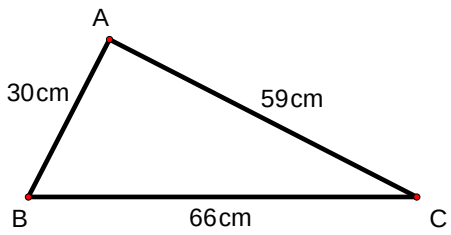
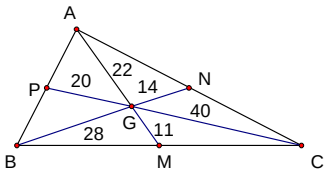
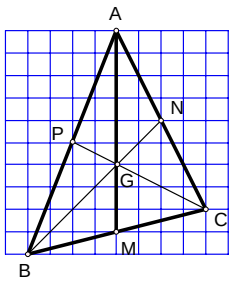
- The mathematics content taught is meaningful;
- The thinking processes of students are transparent through their answers, products, presentation that the viewers can recognize while watching the video.
- The innovation in teaching and learning is discussed, prepared in the mathematics division of the school. Every teacher in the division has his/her own contribution to the innovation.
- The lesson uses the instructional materials that are innovative and appropriate to the school.

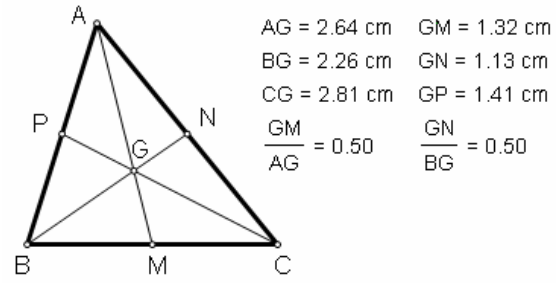
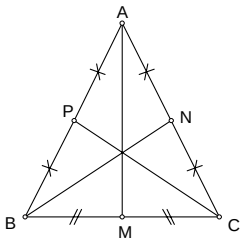
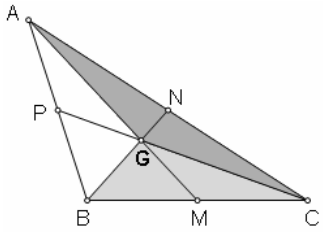
The teacher implements this lesson plan will be chosen by teachers in the division. He has experienced in creating problematic situations and asking open-ended questions that require mathematical thinking of students.

2. Implementing and observing

We implemented the lesson plan designed by Mr. Nguyen Khoa Tu in two different classes before shutting video. In the first class, there were seven observers; they were mathematics teachers of the school. Through discussion, teachers found out that the lesson plan needs to be changed at some points to help students answer the open-ended questions in problematic situation. Some questions was not clear and general so students felt not confident to answer. Some questions required only remember and recall facts and students were not interested in answering the questions. So we revised the lesson plan and taught in another class. In the second class teachers observed students folding, drawing and measuring and found out that students had good responses to questions and actively engaged to the tasks. This time, the teachers agreed that the lesson plan and its lesson flow were suitable to our students at every class grade 7. But the lesson still has something need to be renewed. Then we decided to implement the revised lesson plan the third time in an actual class for shutting video. The students responses to instructional activities at consideration points and evaluation were illustrated in the following table.

Instructional activities	**Students responses to consideration points**
Activity 1 1. How to divide the triangular cake into two equal parts?	**S**: What do you mean by two equal parts? Equal in shape? **S**: I think, their areas are equal, because the amount of cake of each part is the same. **S**: Can we cut the cake by a straight line?
2. Divide an isosceles into two equal parts? 	**S**: It looks easier. I draw the segment from A to midpoint M of BC. And cut through segment AM. **S**: We have two triangle MAB and MAC equal (side – angle – side), so AM is the height. Then S(MAB) = S(MAC) = $\frac{1}{2}AM \times \frac{BC}{2}$. 
3. Divide a right triangle into two equal parts?	**S**: Construct midpoint M of BC. Draw segment AM. Then AB is the height of two triangles MAB and MAC. 

4. General case: Divide an arbitrary triangle two equal parts? *AM* is called the median of triangle *ABC*.	**S**: Now I can see the way to divide the cake. Draw midpoint *M* of *BC*. *AM* will divide the triangle into two equal parts. 
Activity 2 **Q1**. How many medians can you draw in a triangle? Why? **Q2**. What do you mention about the three medians in your figure?	**S**: We always can draw three medians from three vertices. **S**: Three medians are convergent at one point. 
Activity 3. **Task1**. Students are given a triangle drawn on A2 paper without grid.  **Q**. Can you find any ratio of the lengths of segments that determined by the medians.	**S**: We fold *B* to *C* to get midpoint *M*. And then *N* and *P*. **S**: Use ruler to measure the lengths of segments.  *GM* = 11cm; *AG* = 22cm. *GN* = 14cm; *BG* = 28cm. *GP* = 20cm; *CG* = 40cm. **S**: I think *AG* = 2*GM*; *BG* = 2*GN*; *CG* = 2*GP*.
Task 2. **Q**. Can you find any ratio of the lengths of segments that determined by the medians.	**S**: From the grid we can define the midpoints *M*, *N*, *P*. They are the centers of corresponding rectangles. **S**: I see that *AG* = 2*GM*; *BG* = 2*GN*; *CG* = 2*GP*. But I do not know how to prove it. 
Task 3. Open Geometer's Sketchpad, draw a triangle and its three medians. Apply **Measure** | **Length** to measure lengths. Apply **Measure** | **Calculate** to calculate ratio.	**S:** Use GSP to draw a triangle and its medians. Measure lengths and calculate ratio. **S:** Drag point A to change the triangle. Observe the behavior of the ratio. **S**: *AG = 2GM*; *BG = 2GN*. When this group presented their work to the whole class on computer, most of students were surprised because the vertices *A*, *B*, *C* can be dragged but the ratios unchanged.

	
Exercise 1 **Exercise 2** These two exercises aim to consolidate what the students have learnt. The questions are presented on LCD by PowerPoint presentation. (see Lesson Plan in Appendix).	Most of students called upon by teacher answered the questions quite fast. Students showed their understanding and can apply the theory to answer some exercises.
Problem 1. Given an isosceles *ABC*. What is about its three medians? What happens in the special case of an equilateral? 	**S**: An isosceles is symmetry. So I think *BN* = *CP*. **S**: We need to prove *PBC* = *NCB*. **S**: *BC* common, *BP* = *CN*, *PBC* = *NCB*. So *PBC* = *NCB*. Thus *BN* = *CP*. **S**: An equilateral is a special isosceles, so three medians are equal.
Problem 2. Given a triangle *ABC*. Divide the triangle into 3 equal parts? What is about six equal parts? 	**S**: I see that the areas of six small triangles are equal. **S**: S(*MAB*) = S(*MAC*), and S(*MGB*) = S(*MGC*), so taking away the same areas we have S(*AGB*) = S(*AGC*). **S**: Similarly S(*AGC*) = S(*GBC*)

2. Discussing and reflecting

There were thirteen mathematics teachers observed the class including Mr. Tran Du Sinh, Mr. Nguyen Dinh Son (mathematics specialists, Department of Education and Training, Thua Thien Hue), Dr. Tran Vui, Mr. Le Van Liem, Mr. Tran Kiem Minh (Department of Mathematics, Hue University), Mr. Nguyen Huu Bi (The principal), classroom teachers: Mr. Dinh Van Luong (Head of Division), Mr. Nguyen Van Thang, Mr. Tran Van Dien, Mrs. Cao Thi Kim Nhung, Mr. Le Van Cam, Mrs. Tran Thi Thang, Mrs. Nguyen Thi Xuan. After observing the actual class instructed by Mr. Nguyen Khoa Tu, we organized a meeting for sharing ideas and comments. We discussed the following issues. In the

meeting, the teachers gave a lot of comments to four main issues with corresponding questions that were recorded as follows.

1. Lesson study as a means to innovation

In lesson study teachers played a central role to decide what the innovation in teaching and learning is. They are the persons to implement the innovations in their actual classrooms. Teachers help teachers to improve mathematics instruction in the classroom. The innovation can be shared to other teachers.

What was the innovation in teaching method that appeared in the lesson?

We got many answers to this question:

- Lesson started with a real life situation by asking students divide a real cake. The learning process involves with all students working in small groups.
- Students actively sought for and explored mathematical knowledge with the help of teacher.
- Teacher used the way of posing a problem that had the root from real-life situation to make students getting interest at the starting point of the lesson.
- The lesson was student centered, cooperative learning. From a problematic real life situation, teacher facilitated students seek for and construct new knowledge.
- Students actively worked with mathematical problems.
- The lesson is innovative; it is different with old approach of teaching by lecturing.

2. The improvement of students learning

We are seeking for the good practice to improve students learning. Good practice embodied in this lesson study is based on outcomes of successful students learning, including students mathematical thinking, and can be used for further development or challenges.

Was the mathematical content taught in the lesson meaningful and realistic?

- Students understand the relationship between mathematics and real life.
- By folding papers, measuring lengths on papers, students explored the property of three medians. Students gave good comments on the medians of isosceles and equilateral.
- The lesson started from a real situation to develop meaningful mathematics knowledge and then students applied constructed knowledge back to the real life problem.
- Mathematical concept was constructed from a familiar situation of the real world. The knowledge constructed in the lesson helps students solve real life problems.
- Students felt that really have a linkage between mathematics and real life.

How did the key points that intend to enhance students' mathematical thinking show in the lesson?

- Students showed good responses to the questions, but it depends on the ability of each class to have appropriate questions.
- Practicing measurements, inducting from concrete data to generalize the mathematics property.
- The open ended questions gave students chance to explore the property of three medians by themselves.
- Students had good comments on the medians of an isosceles.
- Students understood the way two divide a triangle into three equal parts.

How did the ability of students in responding the questions and tasks requiring mathematical thinking of students show in the lesson?

- The tasks and questions were relevant to students' previous knowledge so they feel confident to seek for new knowledge.
- Most of students apply mathematical reasoning to explain new knowledge they have found.
- Students stated the results explored by themselves accurately.

3. Lesson study supports the professional development

How did the instructional materials support the lesson?

- The instructional materials helped the lesson a lot. They supported students explore and find out new knowledge.
- Low cost instructional materials such as paper, grid paper combined with modern computer helped students explore successfully mathematical ideas.
- Teacher used many kinds of instructional materials that helped students explore corresponding mathematical ideas effectively.
- We need to have an in-service training course for developing instructional materials, especially computer software.

How did the interaction student – student – teacher show in the communication and discussion?

- The students worked in small groups with the guidance, evaluation of teacher. The hint of teacher was effective in discussing between students and students.
- Some students were hesitated and shy to share their knowledge with friends.

What should be changed in the lesson to improve the learning study next time?

- One student should have a separate triangle on paper, so he can fold the paper to explore the property of three medians. After exploring, students discuss in groups to explore the property of centroid.
- This lesson can apply broadly to other classes, but we need to improve the professional ability of teachers and reform the students' assessment.

4. Innovation to the implementation of reform curriculum

How did the thinking process of students show in doing specific mathematical tasks in the lesson that were identified in the reform curriculum?

- Students explore exactly mathematical property of the three medians by observing, folding, measuring and inducing.
- Students can apply what they have learnt to solve some specific problems posed by teacher.
- Most of students showed that they understood the lesson, solved the problems set by the teacher. These problems were revised from the text books.

What is about the application of this lesson plan in the curriculum of lower secondary mathematics?

- With some schools having good facilities such as computer, LCD this lesson plan will be very effective.
- We need to apply this lesson study to other topics and other classes.

- The curriculum is still heavy, a lot of content knowledge in the text book that teachers have to deliver, so the time constraint is a big issue for all students to do the mathematical work by their own paces.
- We need to have relevant facilities in school to prepare appropriate instructional materials for specific topics in the curriculum.
- We need the practical theories that help classroom teachers develop innovation that relevant with the curriculum.

Conclusions

This is the first time we introduced the lesson study cycle in a school. All mathematics teachers in the school agreed that lesson study provides them a good opportunity to see teaching and learning in the classroom scenarios. From that actual scenario teachers develop innovative teaching practices to help students learning. The use of innovation to teaching and learning mathematics in the classroom must be implemented to engage students in meaningful mathematical tasks that require higher order thinking. The innovation provides all students access to a broad range of mathematical ideas. Specifically, the research sought to find answers to the research questions:

1. Lesson study guides teachers to focus their discussions on getting the effective innovation through the cycle. By discussing and sharing new ideas on innovation, observing what happens in actual classroom, teacher improves their teaching and enhances the students learning. We can apply lesson study to many topics in the curriculum. Lesson study as a means to innovation actively affected to teaching and learning mathematics in the school.
2. The innovation as a product of the lesson study helps students have better and meaningful understanding of difficult mathematical concepts. Students were able to discuss and interact freely with their pairs/groups while answering open-ended questions relevant to them. The students communicated friendly their mathematical thinking while they are engaged in the mathematics activities. With hand-on activities, students always have something to share with their friends about problems involving with mathematical thinking.
3. The lesson study for good practice in teaching and learning mathematics actually supported the professional development of teachers. Teachers learnt some things new from their peers and can apply them to the teaching mathematics.
4. The reform mathematics curriculum requires students learning mathematics in an active way to enhance mathematical thinking, so the innovation in teaching and learning mathematics can help teachers implement effectively the curriculum.

Acknowledgement. This lesson study was conducted in Hue City, Vietnam under the collaborative framework involving mathematics education among the APEC Member Economies. Special thanks due to the principal, mathematics teachers of the lower secondary school Nguyen Tri Phuong, Hue City, Vietnam for their contribution to the research.

Reference

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2. Akihiko Takahashi and Makoto Yoshida (2006). *Developing good mathematics teaching practice through lesson study: A U. S. perspective.* Paper presented at APEC-Tsukuba International Conference, January 2006, Tokyo, Japan.
3. Catherine Lewis (2006). *Professional development through lesson study: Progress and Challenges in the U.S.* Paper presented at APEC-Tsukuba International Conference, January 2006, Tokyo, Japan.
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Appendix

LESSON PLAN

Mathematics Lesson Grade 7
Teacher: Mr. Nguyen Khoa Tu, Senior Teacher, Nguyen Tri Phuong Lower Secondary School, Hue City, Vietnam.
Students' ages: 12 years old.
Research Theme: Examining instruction that will help students have the relations between their own experience in dividing a triangular cake into two equal parts and the median, explore the property of three medians and the corresponding ratios by practicing and answering open-ended questions.
Section: 53 in Vietnamese mathematics grade 7 textbook (45 minutes). Topic: *The properties of three medians in a triangle*.

1. Objectives

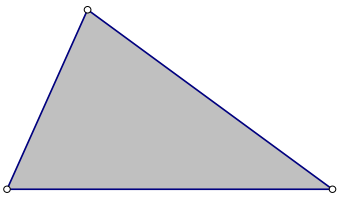
- Students grasp the concept of the medians, centroid of a triangle.
- Understand the convergence of three medians, the property of the centroid through practical works, measuring, drawing and folding papers.
- Know how to draw a median of a triangle, gain skills in using properties of triangle to solve some simple exercises, problems.
- Through the lesson, the teacher creates problematic situations and poses the open-ended questions to enhance students' critical and creative thinking.

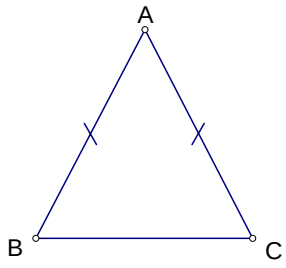
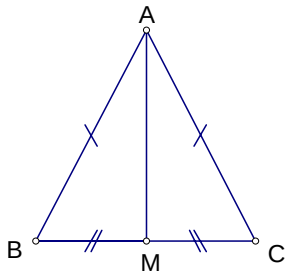
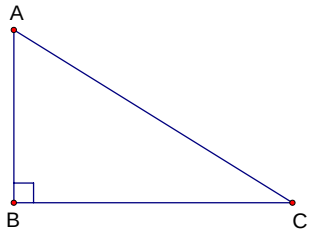
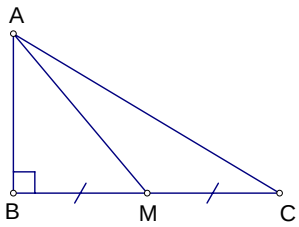
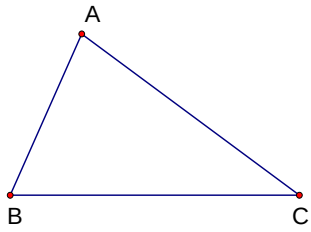
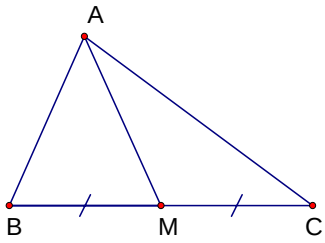
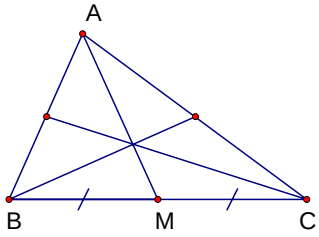
2. Preparation

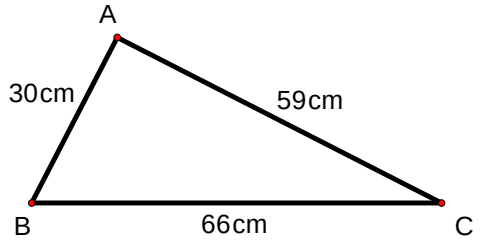
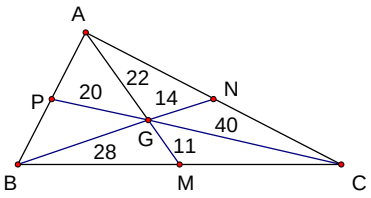
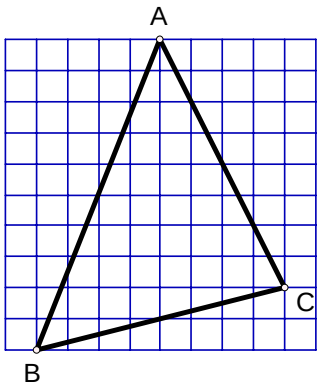
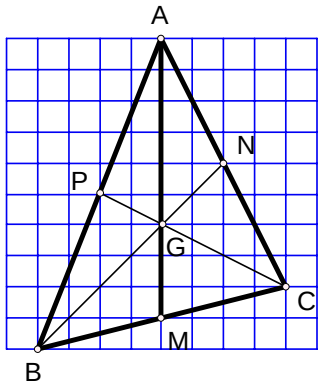
Students: Rulers, compasses, pencils, transparency papers.
Teachers:

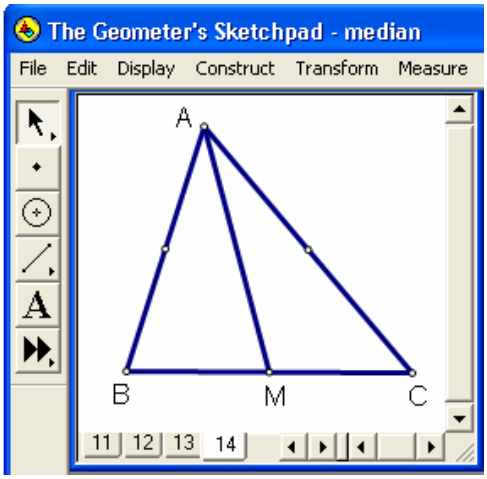
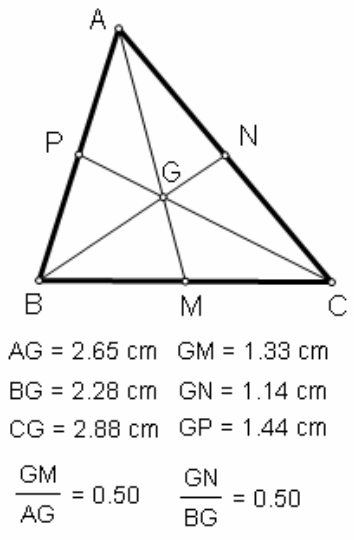
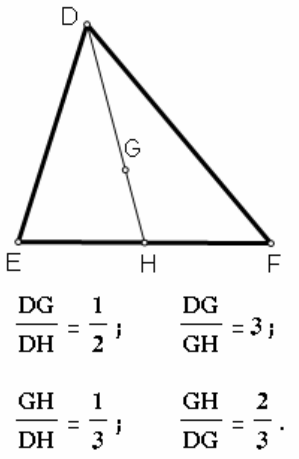
- 2 different triangles drawn on A2 paper.
- 3 different triangles drawn on grid A2 paper.
- One triangle drawn in GSP software.
- The PowerPoint file of the lesson plan, LCD.
- Overhead projector and transparency papers.

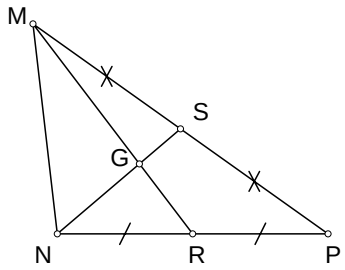
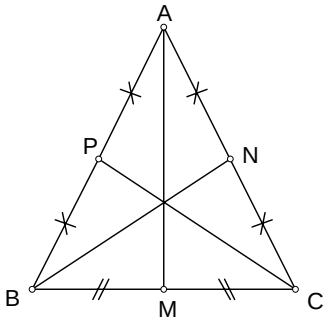
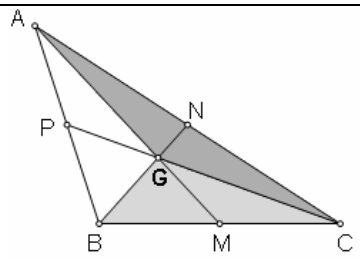
3. Flow of the lesson

Content	*Instructional Activities*	*Points of Consideration & Evaluation*
1. **Introduction Activity** Getting students familiar with the new concept of the median.	**Activity 1**. - Teacher gives students a real triangular cake. Teacher asks students how to divide the cake into two equal parts.  - Teacher asks students to start with two special cases on the board.	Students show their own experience on two equal parts and the area of a triangle. Their areas are equal.
	Isosceles:	S: Draw the segment from A to midpoint M of BC.

		 Show that: S(*MAB*) = S(*MAC*)
	Right triangle: 	Similarly, 
	General case: Arbitrary triangle 	Generally, 
2. **Understand** concept of the median, and the procedure to draw a median of a triangle.	The segment *AM* is called the median of the triangle *ABC*. The procedure to draw a median of a triangle. **Activity 2**. **Q1**. How many medians can you draw in a triangle? Why? **Q2**. What do you mention about the three medians in your figure?	Each student draws a triangle on a piece of paper. Then draw three medians.  The three medians are convergent at one point.
3. **Explore** the property of three medians	**Activity 3**. Students are divided into 6 small groups. Two groups have the same task. Each group works on one task. Which group has good answer will present to the whole class for discussion. **Task1**. Students are given a triangle drawn on A2 paper without grid.	By having students engage in folding paper, drawing, measuring three medians. Three medians are convergent at point *G*.

	 By folding the edges, determine the midpoint of each leg. Draw three medians of the triangle, measure their lengths. **Q1**. What is the relation between three medians? **Q2**. Can you find any ratio of the lengths of segments that determined by the medians?	 Using ruler to measure: *GM* = 11cm; *AG* = 22cm. *GN* = 14cm; *BG* = 28cm. *GP* = 20cm; *CG* = 40cm. Conclusion: *AG* = 2*GM* *BG* = 2*GN* *CG* = 2*GP*
	Task 2. Students are given a triangle drawn on A2 grid paper. Grid 5cm×5cm. Determine the midpoint of each leg. Draw three medians of the triangle, identify their lengths.  **Q1**. What is the relation of three medians? **Q2**. Can you find any ratio of the lengths of segments that determined by the medians? **Theorem:** In a triangle three medians are convergent at centroid, and the length from centroid to a vertex is $\frac{2}{3}$ of the median passing through that vertex.	By having students engage in determining the midpoint of each edge, drawing three medians and identifying the lengths of segments that determined by the medians. Three medians are convergent at point *G*.  *AG* = 2*GM* *BG* = 2*GN* *CG* = 2*GP*
	Task 3. This task will be used only in the class that has computer and LCD projector. Open Geometer's Sketchpad; draw a triangle and its three medians. Apply	Students use GSP to draw a triangle and its medians. Measure lengths and calculate ratio. Drag point *A* to change the

	Measure | **Length** to measure lengths. Apply **Measure** | **Calculate** to calculate ratio.  **Q1**. What is the relation of three medians? **Q2**. Can you find any ratio of the lengths of segments that determined by the medians?	triangle. Observe the behavior of the ratios. Conclusion. 
4. **Consolidate** the theorem.	**Exercise 1** Let *G* be the centroid of triangle *DEF* with the median *DH*. In the following statements which is correct?  	Students apply what they have explore to choose the correct statement: $\frac{DG}{DH}=\frac{2}{3}; \frac{DG}{GH}=2;$ $\frac{GH}{DH}=\frac{1}{3}; \frac{GH}{DG}=\frac{2}{3}.$
	Exercise 2 Given the figure below. Fill in the blanks to have correct equations.	Students recognize different ratios can be gain from the medians and centroid. $MG = \frac{2}{3}MR;$ $GR = \frac{1}{3}MR;$ $GR = \frac{1}{2}MG;$

	 a. *MG* = . . . *MR*; *GR* = . . . *MR*; *GR* = . . . *MG* b. *NS* = . . . *NG*; *NS* = . . . *GS*; *NG* = . . . *GS*.	$NS = \frac{3}{2}NG$; $NS = 3GS$; $NG = 2GS$.
6. **Problem solving**	**Problem 1** Given an isosceles *ABC*. What is about its three medians? What happens in the special case of an equilateral? 	By having students engage in reasoning, making conjecture that *BN* = *CP*. And then prove it. Consider two triangles *PBC* and *NCB*. *BC* common, *BP* = *CN*, *PBC* = *NCB*. So *PBC* = *NCB*. Thus *BN* = *CP*. In an equilateral, three medians are equal.
	Problem 2 Given a triangle ABC. Divide the triangle into 3 equal parts? What is about six equal parts?	 Since S(*MAB*) = S(*MAC*), and S(*MGB*) = S(*MGC*), thus S(*AGB*) = S(*AGC*). Similarly S(*AGC*) = S(*GBC*)