PROFESSIONAL DEVELOPMENT: AN AUSTRALIAN CASE: THE POWER OF ONE-TO-ONE ASSESSMENT INTERVIEWS

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In this paper, I outline what I see as the benefits to teachers' professional development of the use of task-based, one-to-one assessment interviews with students of early and middle years mathematics. I draw upon data from the Victorian Early Numeracy Research Project, our recent work in the domain of rational numbers, and examples from interviews with students in USA and Australia. Such interviews enhance knowledge of individual and group understanding of mathematics, and assist teachers in lesson planning and classroom interactions as they gain a sense of typical learning paths. I argue that an appropriate prelude to lesson study is gaining data on what students know and can do in particular mathematical domains (individually and in a group sense). Large-scale collection of data of this kind also has potential to inform curriculum policy and guidelines.

Background

In the last twenty years, assessment in the early and middle years of schooling has been characterised by a shift in the balance between the summative and formative modes. The inadequacy of a single assessment method administered to students at the end of the teaching of a topic is widely acknowledged. It is increasingly the case that teachers and school administrators regard the major purpose of assessment as supporting learning and informing teaching.

Other reasons for an expansion in assessment methods include a broadening of those skills and understandings which are valued by teachers, schools and educational systems. For example, in the publication, Adding It Up (Kilpatrick, Swafford, & Findell, 2001), the term "mathematical proficiency" was introduced, which the authors saw as including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The limitations and disadvantages of pen and paper tests in gathering accurate data on children's knowledge were well established by Clements and Ellerton (1995). They contrasted the quality of information about students gained from written tests (both multiple-choice and short-answer) with that gained through one-to-one interviews, and observed that children may have a strong conceptual knowledge of a topic (revealed in a one-to-one interview) but be unable to demonstrate that during a written assessment.

The findings of this research contrast with the continued common emphasis in many classrooms today on procedural fluency. Reading issues in written tests are also of great significance.

For the past fifteen years, it has become common for teachers of literacy to devote time to assessing students individually, and using the knowledge gained to teach specific skills and strategies in reading (Clay, 1993; Hill & Crevola, 1999). The late 1990s, in Australia and New Zealand, saw the development and use of research-based one-to-one, task-based interviews on a large scale, as a professional tool for teachers of mathematics (Bobis, Clarke, Clarke, Gould, Thomas, Wright, & Young-Loveridge, 2005).

I outline below examples from two projects and the experiences of the authors in developing, piloting, and using interviews within professional development contexts. The potential of such interviews for enhancing teacher content knowledge and knowledge for teaching (Hill & Ball, 2004) is discussed. It will be argued that the use of such interviews can enhance many aspects of teacher knowledge, with consequent benefits to students.

The Early Numeracy Research Project

The Early Numeracy Research Project (ENRP) research and professional development program conducted in Victoria from 1999 to 2001 in Years Prep to 2 (with some limited data collection of the original Prep cohort in Years 3 and 4, in 2002 and 2003 respectively), investigated effective approaches to the teaching of mathematics in the first three years of schooling, and involved teachers and children in 35 project (“trial”) schools and 35 control (“reference”) schools (Clarke, 2001; Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). In all, the project involved 353 teachers and over 11 000 students of ages 4 to 8.

There were three key components to this research and professional development project:

- the development of a research-based framework of “growth points” in young children’s mathematical learning (in Number, Measurement and Geometry);
- the development of a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively); and
- extensive professional development at central, regional and school levels, for teachers, coordinators, and principals.

As part of the ENRP, it was decided to create a framework of key “growth points” in numeracy learning. Students’ movement through these growth points in trial schools, as revealed in interview data, could then be compared to that of students in the reference schools. The project team studied available research on key “stages” or “levels” in young children’s mathematics learning (e.g., Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; Mulligan & Mitchelmore, 1996; Owens & Gould, 1999; Wilson & Osborne, 1992; Wright, 1998; Young-Loveridge, 1997), as well as frameworks developed by other authors and groups to describe learning.

The decision was taken to focus upon the strands of Number (incorporating the domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies), Measurement (incorporating the domains of Length, Mass and Time), and Geometry (incorporating the domains of Properties of shape, and Visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There were typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children "count-all" to find the total ("1, 2, 3, ..., 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children's developing understanding of Addition.

These growth points informed the creation of interview tasks, and the recording, scoring and subsequent data analysis, although the process of development of interview and growth points was very much a cyclical one. In discussions with teachers, I have come to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of mapping of the conceptual landscape. However, I do not claim that all growth points are passed by every student along the way.

The one-to-one interview was used with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 50-minute period, depending upon the interviewer's experience and the responses of the child. The interviews were conducted by the classroom teacher in trial schools, and a team of interviewers in reference schools. A range of procedures was developed to maximise consistency in the way in which the interview was administered across the 70 schools.

Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child moved through all of these. The interviewer made a decision after each task. Given success, the interviewer continued with the next task in the domain as far as the child could go with success. Given difficulty with the task, the interviewer either abandoned that section of the interview and moved on to the next domain or moved into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

The interview provided information about growth points achieved by a child in each of the nine domains. Below are two questions from the interview. These questions focus on identifying the mental strategies for subtraction that the child draws upon. The strategies used were recorded on the interview record sheet.

> 19) Counting Back
> For this question you need to listen to a story.
> a) Imagine you have 8 little biscuits in your play lunch and you eat 3.
> How many do you have left? ... How did you work that out?
> *If incorrect answer, ask part (b):*
>
> b) Could you use your fingers to help you to work it out? *(It's fine to repeat the question, but no further prompts please).*
>
> 20) Counting Down To / Counting Up From
> I have 12 strawberries and I eat 9. How many are left? ... Please explain.

It was intended that the interview would provide a challenge for all children. Over 36,000 interviews were conducted by teachers and the research team during the ENRP, and only one child was successful on every task — a Grade 2 boy in the second year of the project. It appeared that the aim of challenging all was achieved, with one possible exception!

Australian Catholic University Rational Number Interview

Following the perceived success of the Early Numeracy Research Project, it was decided to develop a one-to-one interview for teachers of nine- to fourteen-year olds. Given the recognised difficulty with fractions and decimals for many teachers and students (see, e.g., Behr, Lesh, Post, & Silver, 1983; Kieren, 1988; Lamon, 1999; Steinle & Stacey, 2003), it was decided to make rational numbers the focus of the interview. Anne Roche adapted and developed tasks in decimals (see, e.g., Roche, 2005; Roche & Clarke, 2004) and Annie Mitchell in fractions (see Mitchell & Clarke, 2004; Mitchell, 2005). In 2005, Clarke, Roche and Mitchell collaborated with Jan Stone (Association of Independent Schools, New South Wales) and Professor Richard Evans (Plymouth State University) in refining these tasks. A major source of tasks included the Rational Number Project (Behr & Post, 1992).

Once again, the selection of tasks used by the teacher is made during the interview, according to students' responses. There are currently 31 tasks assessing fraction understanding, 14 assessing decimal understanding, and 3 assessing proportional reasoning. Development on a range of tasks for percentages is continuing. To this point, approximately 70 teachers have been involved in piloting the tasks with their students. Two sample tasks are given in Figure 1.

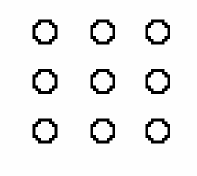
Nine dots	**Ordering**
Show the student the picture of 9 dots.  If this is three-quarters of a set of dots, how many dots is two-thirds of the set? *(drawing is okay if necessary) ……….* Please explain your thinking. [adapted from Cramer & Lesh, 1988]	*Place the cards randomly on the table.* Put these numbers in order from smallest to largest. *Encourage the student to think out loud while ordering them* **a) 0 0.01 0.10 .356 0.9 1 1.2** *Show each card below in turn* **1.70 1.05 .10** b) Where would this decimal go? Why does it belong there?

Figure 1. Sample tasks from the Australian Catholic University Rational Number Interview.

It should be noted that the task of developing "growth points" or a learning and assessment framework in rational number understanding is proving more elusive than for the domains of the ENRP. At present, our compromise is a statement of 25 "big ideas" in rational number knowledge, skills and understanding.

For example, one big idea is “works within a variety of physical and mental models (areas and regions, sets, number lines, ratio tables, etc.), in continuous and discrete situations.” However, because the domain of rational numbers is made up of many aspects or “subconstructs” (Kieren, 1988), and the use of many models within each subconstruct (Lamon, 1999), it has been a challenging task to try to map out a “conceptual landscape” for this content.

Similarly, it has been difficult to arrange the interview tasks in the same way as the ENRP, with many “drop-out points” and detours, as I have found that success or lack of success on a given task is not necessarily a good predictor for performance on another task, even when they seem closely related.

In the following sections, particular tasks and insights from teachers will be used to build the argument of the power of the interview as a professional development tool. I will outline the benefits to teacher professional growth and therefore the quality of teaching of the use of task-based, one-to-one interviews by mathematics teachers in the early and middle years of schooling.

INTERVIEWS AS A POWERFUL TOOL FOR MATHEMATICS TEACHERS

In the remainder of the paper, I will use data collected from teacher surveys as supporting data, and anecdotes from our own experience, a combined total of approximately 500 interviews.

Higher quality assessment information

In contrast to the traditional pen and paper test, a carefully-constructed and piloted one-to-one interview can provide greater insights into what students know and can do. Student strategies are recorded in detail on the interview record sheet. For example, in addition and subtraction, for the two subtraction tasks outlined earlier in this article, the teacher completes the record sheet, as shown in Figure 2, recording both the answer given and the strategies used. The emphasis on recording both answer and strategies is clear recognition that the answer alone is not sufficient.

The act of completing the record sheet requires an understanding of the strategies listed (e.g., modelling all, fact family, count up from, etc.). The use of the interview is therefore building pedagogical content knowledge (Shulman, 1987).

The capacity of the teacher to take the information on the record sheet and “map” student performance in relation to the growth points or “big ideas” is a key step in the process. Teachers after conducting the interview are likely to ask the reasonable question in relation to planning, “So now what?” If they have a clear picture of individual and group performance in particular mathematical domains, they are then in a position, hopefully with support of colleagues, to plan appropriate classroom experiences for individuals and groups.

19. Count Back / Modelling All (8 – 3)

a. Answer .... ❒

- ***Known fact or fact family (eg., 5 + 3 = 8)***
- ***Count back all, in head (7, 6, 5 or 8, 7, 6, 5)***
- ***Count back all, with fingers used to keep track only (7, 6, 5 or 8, 7, 6, 5)***
- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

b. Answer .... ❒

- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

20. Count Down To / Count Up From (12 – 9)

Answer .... ❒

- ***Known fact or fact family (eg., 9 + 3 = 12)***
- ***Count down to (12, 11, 10, 9)***
- ***Count up from (9, 10, 11, 12)***
- ***Fingers used during "count down to" or "count up from" to keep track only***
- Count back all (12, 11, 10, 9, 8, 7, 6, 5, 4, 3)
- Modelling all (shows 12 "things", then takes away 9 "things", leaving 3)
- Other ....

Figure 2. An excerpt from the addition and subtraction interview record sheet.

A focus on mental computation

Northcote and McIntosh (1999), using surveys of all reported computations of 200 adults over a 24-hour period, concluded that approximately 83% of all computations involved mental methods, with only 11% involving written methods. In addition, they found that over 60% of all computations only involved an estimate. These findings influenced greatly the construction of our interviews, where mental computation and estimation feature prominently.

Physical involvement: Making the task match the desired skill

Some mathematical skills and understandings can be very difficult to assess without some kind of physical task. As one teacher wrote, "to see whether children can do physical things, we sometimes need to watch." Consider this task from the Place Value section of the ENRP interview. The child is given a pile of icy-pole sticks, 7 bundles of 10 sticks each wrapped in an elastic band and about 20 loose ones. The teacher explains to the child that there are "bundles of ten and some more loose ones." The child is then shown a card with the number "36" on it, and asked to "get this many icy-pole sticks."

The student response is a very helpful indicator of place value understanding, in that some will feel the need to pull apart the bundles of ten (possibly indicative of an understanding of 36 only as the number in the sequence 1, 2, …, 36); some will count "10, 20, 30," and then "1, 2, 3, 4, 5, 6." A subtle improvement is the child who is able to say "3 of these and 6 of those," without any need to count. It is difficult to imagine a task that didn't involve this level of physical action providing the same opportunity for the teacher to gain what they do from listening and watching.

Objects of various kinds also increase the level of accessibility to tasks, and enjoyment of the experience for the student. There is also a number of topics in the mathematics curriculum which are not easily assessed by traditional means, e.g., visualisation and orientation, and manipulation of objects allowed students to show what they know.

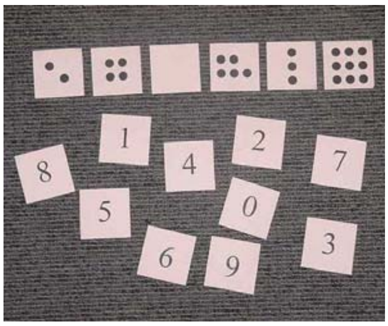
Large scale valid and reliable data

Processes used by the research team to maximise reliability and validity of interview data have been detailed elsewhere (see Clarke, 2001, Clarke et al., 2002). Having data on over 36 000 ENRP interviews across Grades Prep to 4 (the project focused on Grades Prep to 2, but a small "spin-off" project involved interviews with over 1000 students at each of Grades 3 and 4), provided previously-unavailable high quality data on student performance. These data had several benefits:

- Information for teachers on what "typical performance" for various grade levels looked like enabled them to relate the performance of their students to that of the cohort. For example, Table 1 shows the percentage of children on arrival at schools in trial schools who were able to match numerals to their corresponding number of dots. The data on children in the first year of school is discussed in considerable detail in Clarke, Clarke and Cheeseman (2006).

  Table 1. Performance of trial school children on entry to school in February 2001 on selected tasks (%) (*n* = 1437)

	Percent Success
Match numeral to 2 dots	86%
Match numeral to 4 dots	77%
Match numeral to 0 dots	63%
Match numeral to 5 dots	67%
Match numeral to 3 dots	79%
Match numeral to 9 dots	41%

  Teachers and researchers found considerable variation within classes in what students knew and could do, to an even greater extent than many previously thought. Of course, this makes a mockery of arguments that "all Prep children should be studying this and not that."

- Information is available for state departments of education and curriculum developers to inform their work. One of the most powerful pieces of data which is hopefully informing the development of the Victorian Essential Learning Standards (Victorian Curriculum and Assessment Authority, 2005) is found in the domain of Addition and Subtraction. Achievement of the growth point "Derived strategies in addition and subtraction" was assessed by the following tasks:

12 - 6   7 + 8   19 - 15   16 + 5   36 + 9

- Students were deemed to have achieved the growth point if they answered correctly (mentally, with no time limit), and used at least three preferred strategies across the five problems. For example, for 36 + 9, counting by ones (“36, 37, 38, …, 45”) is a non-preferred strategy, while 36 + 10 - 1 would be a preferred strategy.

  At the end of Grade 2, only 19% of “typical children” could succeed on this basis. Even in trial schools (where teachers had been given intensive professional development), the percentage was only 31%. Yet, at the time, the state curriculum guidelines implied that virtually all children should be able to do these tasks. In light of these data (and the figure for typical students at the end of Grade 4—55%), it would appear that the state curriculum needs revision in terms of this content, as well as a consideration of whether the common practice of introducing conventional algorithms as early as Grade 2 is completely inappropriate (see Clarke, 2005 for more on this issue).

Building a knowledge of variations in performance across grade levels

It is interesting to collect sufficient data in order to observe trends in development of student understanding across the grade levels. To illustrate this point, a task adapted from the Rational Number Project (Cramer, Behr, Post & Lesh, 1997; Cramer & Lesh, 1988) and used in our Rational Number Interview, is shown in Figure 3.

Figure 3. A task used in the Australian Catholic University Rational Number Interview.

Fraction pie

Show the student the pie diagram.

a) What fraction of the circle is part B?.....
How do you know that?
b) What fraction of the circle is part D?.....
How do you know that?

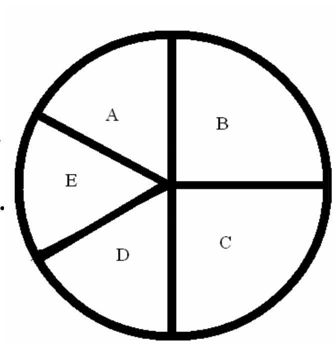


Table 2 shows student performance by grade level on the two parts of this task. To be correct, both the correct answer and an appropriate explanation were required. Students who were unsuccessful on part (a) were not given part (b) to attempt. Once again, the difficulty posed by this task for many students, possibly due to a lack of familiarity with tasks where not all parts are the same size, has implications for both emphasis and the pace of moving through content in fractions.

Table 2. Student Performance on Part-Whole Task (Continuous Case) by Grade Level (Years 4-6)

Q4 Part B Pie			
Grade	4	5	6
Correct	35/58	52/68	50/61
%	60%	76%	82%

Q4 Part D Pie			
Grade	4	5	6
Correct	29/58	36/68	33/61
%	50%	53%	54%

One of the advantages of administering the assessment interview at both the beginning and end of the school year was that teachers were provided, face-to-face, with exciting evidence of growth in student understanding over time.

Relating performance in one part of the interview to performance in another part

It is informative for teachers and researchers to consider whether understandings evident in one part of the interview prove accessible in another context. A major feature of teaching for relational understanding (Skemp, 1976) is that understanding enhances transfer (Hiebert & Carpenter, 1992). Among the possible tasks a student might encounter in the ENRP interview Counting section were tasks asking them to count by 2s, 5s, and 10s from 0. Given success, they counted by 10s and 5s, from 23 and 24 respectively.

In the Multiplication and division part of the interview, as part of a task assessing what we called abstracting multiplication and division (see Sullivan, Clarke, Cheeseman, & Mulligan, 2001), students were shown an array of dots which was then partially-covered as shown in Figure 4. They were then asked: “How many dots altogether on the card?” Even when students who counted by ones were prompted by, “could you do it a different way, without counting them by ones,” the success rate was not high. Only 37.5% of 2942 Year Prep to 2 students were successful in transferring those skills to this new context.

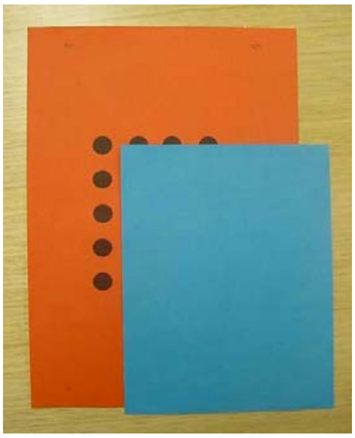


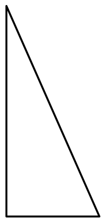
Figure 4. An array task.

Enhanced teacher knowledge of mathematics

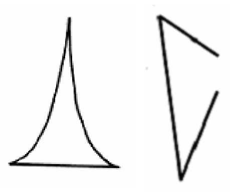
Our experience in working with teachers is that the use of the interviews enhances teacher content knowledge. In the middle years, many teachers acknowledge their lack of a connected understanding of rational number, often using limited subconstructs (sometimes only part-whole), and limited models (such as the ubiquitous “pie”). Many teachers have reported that their own understanding of rational number (e.g., an awareness of subconstructs of rational number such as measure and division and the distinction between discrete and continuous models) has been enhanced as they observe the variety of strategies their students draw upon in working on the various tasks and complete the record sheet.

Some might presume that teacher content knowledge is not an issue. However, many teachers reported that terms such as "counting on," "near doubles", and "dynamic imagery" were unfamiliar to them, prior to their involvement in the ENRP. It is interesting to consider whether this is content knowledge or "knowledge for teaching" (see, e.g., Ball & Bass, 2000; Ball & Hill, 2002; Hill & Ball, 2004).

Teachers develop an awareness of the common misconceptions and strategies which they may not currently possess



As teachers have the opportunity to observe and listen to students' responses, they become aware of common difficulties and misconceptions. For example, many children in Years Prep to 4 were unable to give a name to the shape on the left. It wasn't expected that they would name it "right-angled triangle," but simply "triangle". Because it didn't correspond to many students' "prototypical view" (Lehrer & Chazan, 1998) of what a triangle was (*a triangle has a horizontal base and "looks like the roof of a house"—either an isosceles or equilateral triangle*), some called it a "half-triangle, because if you put two of them together you get a real triangle." Many students nominated the two shapes on the right as triangles.



It was clear from a teaching perspective that it was important to focus on the properties of shapes, and to present students with both examples and non-examples of shapes.

The quiet achievers sometimes emerge

> *In every class there is that quiet child you feel that you never really 'know'—the one that some days you're never really sure that you have spoken to. To interact one-to-one and really 'talk' to them showed great insight into what kind of child they are and how they think (ENRP teacher, March 1999, quoted in Clarke, 2001).*

In response to a written question on highlights and surprises from the Early Numeracy Interview, a number of teachers noted that the one-to-one interview enabled some "quiet achievers" to emerge, and several noted that many were girls. There appeared to be some children who didn't involve themselves publicly in debate and discussion during whole-class or small-group work, but given the time one-to-one with an interested adult, really showed what they knew and could do.

> *The greatest highlight was that no matter at what level the children were operating mathematically, all children displayed a huge amount of confidence in what they were doing. They absolutely relished the individual time they had with you; the personal feel, and the chance to have you to themselves. They loved to show what they can do (ENRP teacher, March 1999, quoted in Clarke, 2001).*

Improved teacher questioning techniques (including the use of wait time)

Teachers noted that the interview provided a model for classroom questioning, and as a result of extensive use of the interview, they found themselves making increasing use of questions of the following kind:

- Is there a quicker way to do that?
- How are these two problems the same and how are they different?

- Would that method always work? . . .
- Is there a pattern in your results?

Teachers also observed the power of *waiting* for children's responses during the interview, noting on many occasions the way in which children who initially appeared to have no idea of a solution or strategy, thought long and hard and then provided a very rich response. Such insights then transferred to classroom situations, with teachers claiming that they were working on allowing greater wait time.

Tasks provide a model for classroom activities

Teachers were strongly discouraged from "teaching to the test" through presenting identical tasks to those in the interview during class. Nevertheless, the tasks did provide a model for the development of different but related classroom activities. For example, in the Place Value section of the Early Numeracy Interview, students are asked to type numbers on the calculator as they are read by the teacher or read numbers that emerge as they randomly pick digits and extend the number of places (ones, tens, hundreds, etc.) of the number on the screen.

Seeing the potential of the calculator as a tool for exploring and extending place value understanding, teachers would try tasks such as "type the largest number on the calculator which you can read (but no zeros in it)." The reason for the instruction to have no zeros in the number was because some children will be able to read a million, but not necessarily 386. Such a task provides an opportunity for the teacher to challenge them to make the number even larger. This task, re-visited regularly, provides a helpful measure of growth in student understanding over time, and therefore can be used as an ongoing assessment tool.

Teacher professional growth: Some final comments

At a professional development day involving all 250 or so teachers) towards the end of 1999, ENRP teachers were asked to identify changes in their teaching practice (if any), as a result of their involvement in the project. There were several common themes, many of which can be related to the professional growth experienced through the use of the interview:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on "pulling it together" at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and "real life mathematics".
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Several of the themes discussed in this article are evident in the following quote from a teacher who attended the professional development program:

> *The assessment interview has given focus to my teaching. Constantly at the back of my mind I have the growth points there and I have a clear idea of where I'm heading and can match activities to the needs of the children. But I also try to make it challenging enough to make them stretch.*

ONE –TO-ONE INTERVIEWS AND LESSON STUDY

So what is the potential relationship between the use of one-to-one assessment interviews and lesson study? In describing the Early Numeracy Research Project, we have sometimes used these words: "understanding, assessing and developing young children's mathematical thinking."

The growth points provide a way of *understanding* students' thinking and possible pathways or trajectories through which students might move, the interview provides a way of establishing where students "are at" in relation to these pathways (*assessing*), and the professional development program provided an opportunity to explore how this understanding might be developed further ("*developing*"). I would argue that lesson study fits very nicely in with the third aspect. If teachers have a clear picture of their students' understanding of mathematics and a framework against which this can be mapped, then lesson study provides an ideal model for planning "where to from here?" In this way, the use of one-to-one assessment interviews is in complete harmony with the lesson study approach.

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