APEC INTERNATIONAL SYMPOSIUM

INNOVATION AND GOOD PRACTICES FOR TEACHING AND LEARNING MATHEMATICS THROUGH LESSON STUDY

MATHEMATICS EDUCATION FOR THE KNOWLEDGE - BASED SOCIETY

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1. What am I offering in this address?

It is a great honour for me to give the opening address to this conference and of course I am very happy to be here again in Khon Kaen, Thailand. I am also happy with the topic which I have been given by the organisers, and my talk today will offer the following five contexts for you, which I will briefly clarify now:

- A frame for the conference discussions?

This conference is focussed on teacher education in mathematics and particularly on the use of 'lesson study' as a means for developing both the theory and the practice of mathematics teacher education. But it is necessary to keep this topic framed, particularly in such a short conference as this is, in order that we maximise our time together.

- A context for considering generalisations?

Mathematicians and mathematics educators love generalising – it is valued as one of the basic means for developing mathematical ideas. The challenge for us however is that where mathematics seeks to develop ever more abstract ideas, teacher education must always strike a balance between abstract theory and concrete practice.
Both student teachers and experienced teachers will reject any ideas for teacher education that do not strike what they feel is the right balance between the two objectives.

- An explicitation of some hidden assumptions?

In my research on values in mathematics education, it is clear that most values teaching and learning takes place implicitly in the mathematics classroom. This is also likely to be the case in the context of this project, which is even more problematic since we come from very different cultural and social contexts. It is vital that in our discussions we keep aware of the hidden assumptions and values which are not necessarily shared by all.

- A personal view on the values involved in this project?

Having mentioned values above, it is necessary for me also to clarify my values and assumptions within this conference topic. No researcher is value-free!

- An opening up of some of the issues involved?

Although my topic is not especially about lesson study, nevertheless I feel it is necessary for me to at least expose my ideas about some of the issues involved in this development. (I must also ensure of course that you do not go to sleep!)

2. Definition of knowledge-based society

My topic is certainly an interesting one, full of issues of definition, values, goals and predictions. But in 2003 there took place the World Science Forum in Budapest, Hungary, and their theme for that conference was Knowledge and Society (see website ref.) In it they gave a useful definition of a Knowledge-based society, and here are the main points:

- A knowledge-based society is an innovative and life-long learning society
- It possesses a community of scholars, researchers, technicians, and firms engaged in research and in production of high-technology goods and service provision.
- It forms a national innovation-production system, which is integrated into international networks of knowledge production, diffusion, utilization, and protection.
- Its communication and information technological tools make vast amounts of human knowledge easily accessible.
- Knowledge is used to empower and enrich people culturally and materially, and to build a sustainable society.
- Innovative
- Life-long learning
- National and international networks of learning communities
- ICT goods and service provision
- Empowerment/enrichment of society culturally and materially
- A sustainable society

In some ways this is a formidable list, containing both descriptive and prescriptive ideas. Every country would have something to aspire to from this list and all of us attending this conference here today would have reservations about whether our countries are achieving any of these goal descriptions. But it is good to have such a challenging list to begin our deliberations here.

3. How to consider education in this new context?

In particular it is a challenge to consider education within this new context. But is a knowledge-based society really a new idea? We should ask ourselves what is different now. Society has always used and taught knowledge, but originally it was the family context which provided the education, from whom the knowledge came and with the elders being the 'teachers'. Gradually as education became more formalised, the schools developed from the families. Also the content of what was taught became more organised, and became based on the knowledge supplied from the 'academy'. Finally the teachers became officially recognised, needing official qualifications and eventually being specifically trained.

Now as the knowledge society is developing, we find that the new knowledge comes from 'outside' the accepted sources, from the Web, from the media, from peer-group networks and also from wide international sources. But many questions also arise for us in education: Whose knowledge is it? Who is producing it? Whose personal knowledge is being exploited and whose personal knowledge is being ignored? Basically the question now facing us is: What is the source of the authority of any new knowledge?

4. Kinds of education => Kinds of mathematics education

Coombs(1985) gave a very helpful analysis in his book ‘The world crisis in education.’ He based his analysis on three kinds of education: formal, non-formal and informal. According to Coombs, there are crucial distinctions to be made between these, and I feel that we too need to be aware of these within our special field. Thus I offer you three kinds of mathematics education whose distinctions are I think crucial in considering our roles in a knowledge-based society. The three sets of characteristics are based on Coombs.

Formal mathematics education is the formal system most of us are part of, and it consists basically of the state system which exists in most countries. It is largely the only kind which gets recognised in research in our field, and operates up to student ages of around 16 or 18 years. It is

- Structured
- Compulsory
- A coordinated system, which is
- Staffed by recognised teachers

Non-Formal mathematics education is the kind of non-compulsory and optional education offered by courses such as for adult education, or vocational education and training. For formal school-age students, it could be after school classes, cram-school classes etc. Generally it is:

- Structured
- Non-compulsory/optional
- With a specific focus
- Coordinated to a certain extent, and
- Some teachers are recognised, some not.

Informal mathematics education is the largely unstructured and often accidental education which comes from a variety of sources, and ‘happens’ to all of us. Whether it is on the Web, on TV, via computer programs, in the papers, or journals, or occurring at conferences like this one. Its characteristics are that it is:

- Unstructured
- Accidental
- Uncoordinated, and with largely
- Unrecognised ‘teachers’

Coombs particular contribution for me was that we have to consider the last category as a form of education, to look at it through educational eyes. It makes us think about questions like Who are the ‘teachers’? What is their agenda? What is the nature of the mathematics being taught? How do these ideas intersect with those being taught in the Formal system?

5. Where is development happening?

If we continue with these three categories, we can ask some more interesting questions, such as where is development happening in mathematics education? Regarding the three categories, we can summarise things this way:

Formal Mathematics Education (FME):

- Developing slowly in terms of mathematical knowledge
- Developing slowly in terms of pedagogy
- Difficult to change the system
- Difficult to change the examinations
- Student input to changes limited

Non-Formal Mathematics Education (NFME):

- More responsive to knowledge changes
- Pedagogical developments less restricted
- More scope for individual teachers to develop courses and materials
- Less controlled by examinations
- More responsive to student inputs as ‘clients’.

In-Formal Mathematics Education (IFME):

- Responsive to, and often initiating, knowledge changes
- Opportunistic with respect to ‘pedagogical’ changes, no examinations
- No formal teachers means greater experimentation and innovation
- Client-led learning
- Lack of control on authority for knowledge

6. Responses of Mathematics Education to the growth of the knowledge-based society

Now we can begin to identify how mathematics education is responding to the growth of the new knowledge based society. For example we can see that IFME is highly responsive, and is often leading the developments. Via the Web, new computer programs, and international networks, we are seeing many developments (or pressures for developments) taking place.

NFME is responding in some ways, in particular in changing the structured courses to respond to client needs in the training and vocational education sectors. In fact as the business models for the NFME providers become much more sophisticated, and in line with other businesses, this sector of mathematics education is exerting much influence on the formal sector. In some ways the borders between IFME and NFME are becoming rather blurred.

On the other hand, and in stark contrast, the FME sector is slow to respond, and even then with minimal changes. There are some changes in curriculum taking place, particularly with the pressures from those who are advocating more emphasis on Numeracy, but there have been few changes in pedagogy, even though ICT is becoming more prevalent in schools and classrooms.

7. What particular developments should we aim for in FME to prepare our students for the Knowledge-based society?

Firstly any Formal Mathematics Education must balance several complementarities:

- Individual growth v. class/group/grade development
- Traditional content v. expanded knowledge
- Traditional pedagogy v. ICT and student-led pedagogical approaches
- Formal systemic examinations v. individual assessment

So bearing these balances in mind, let us explore the definitions of, and criteria for, a knowledge-based society and see how we would develop our FME in our different countries:

Innovative society

- Teaching should encourage more creativity in the students
- Individuals' and groups' original ideas should be valued by teachers
- Assignments should allow creative initiatives
- Assessments should reward creative ideas and solutions to mathematical problems

Life-long learning

- Laying the skill foundations for problem-solving and creativity
- Teaching information searching
- Teaching information validating
- Developing publication and knowledge-sharing skills

National and international networks of learning communities

- Encouraging knowledge networking
- Demonstrating learning community activities
- Contributing to, and using information from, those communities

ICT goods and service provision

- Increasing the familiarity of teachers and students with ICT equipment and software
- Recognising the limitations of ICT as information and communications media

Empowerment/enrichment of society culturally and materially

- Recognising the cultural and historical nature of mathematics knowledge
- Recognising how mathematics assists, informs, and thereby 'formats' society
- Recognising the limitations of mathematical knowledge

A sustainable society

- Mathematics education should embrace environmental education
- Values education should be more explicit
- Balancing individual goals and societal goals should be addressed

8. Final thoughts

Lesson study needs recognising as a socially situated research practice

This is where the Social dimension of mathematics education needs greater recognition (Bishop, 1991). It operates at these five main levels:

Cultural level - language, values, culture, history
Societal level – politics of society, educational institutions,
Institutional level – within institutional rules and goals, internal politics
Pedagogical level – within the classroom, teacher and students as social group
Individual level – individual students' and teachers' backgrounds and goals

Any lesson study research is therefore situated within any particular cultural, societal, and institutional context.

The cultures and values of researchers need recognising

Related to the points above, we should note that no research is ever value free, there are always goals, assumptions, histories and institutional politics at work. Moreover, we researchers are never value free either! We have our own goals, histories and values, and these will inevitably affect what and how we prefer to research.

International sharing, networking and awareness need encouraging

At an international conference such as this, and despite the fact that many people here are working on the same lines, there will inevitably be similarities and differences between us. This should not be considered as a problem but welcomed. We all develop our ideas by experiencing contrasts, and thus we should be celebrating and valuing diversity and enjoying the challenging contrasts such a conference provides. In the same way we should all of us beware of cultural/linguistic imposition. Regrettably I am guilty of imposing my language on you all, and I therefore finish by apologising for that. Nevertheless I hope that you will forgive me, and also that you try to see through the barriers of languages to consider the ideas which I have presented to you.

I hope you all have an enjoyable and stimulating conference.

References

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PROFESSIONAL DEVELOPMENT: AN AUSTRALIAN CASE: THE POWER OF ONE-TO-ONE ASSESSMENT INTERVIEWS

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In this paper, I outline what I see as the benefits to teachers' professional development of the use of task-based, one-to-one assessment interviews with students of early and middle years mathematics. I draw upon data from the Victorian Early Numeracy Research Project, our recent work in the domain of rational numbers, and examples from interviews with students in USA and Australia. Such interviews enhance knowledge of individual and group understanding of mathematics, and assist teachers in lesson planning and classroom interactions as they gain a sense of typical learning paths. I argue that an appropriate prelude to lesson study is gaining data on what students know and can do in particular mathematical domains (individually and in a group sense). Large-scale collection of data of this kind also has potential to inform curriculum policy and guidelines.

Background

In the last twenty years, assessment in the early and middle years of schooling has been characterised by a shift in the balance between the summative and formative modes. The inadequacy of a single assessment method administered to students at the end of the teaching of a topic is widely acknowledged. It is increasingly the case that teachers and school administrators regard the major purpose of assessment as supporting learning and informing teaching.

Other reasons for an expansion in assessment methods include a broadening of those skills and understandings which are valued by teachers, schools and educational systems. For example, in the publication, Adding It Up (Kilpatrick, Swafford, & Findell, 2001), the term "mathematical proficiency" was introduced, which the authors saw as including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The limitations and disadvantages of pen and paper tests in gathering accurate data on children's knowledge were well established by Clements and Ellerton (1995). They contrasted the quality of information about students gained from written tests (both multiple-choice and short-answer) with that gained through one-to-one interviews, and observed that children may have a strong conceptual knowledge of a topic (revealed in a one-to-one interview) but be unable to demonstrate that during a written assessment.

The findings of this research contrast with the continued common emphasis in many classrooms today on procedural fluency. Reading issues in written tests are also of great significance.

For the past fifteen years, it has become common for teachers of literacy to devote time to assessing students individually, and using the knowledge gained to teach specific skills and strategies in reading (Clay, 1993; Hill & Crevola, 1999). The late 1990s, in Australia and New Zealand, saw the development and use of research-based one-to-one, task-based interviews on a large scale, as a professional tool for teachers of mathematics (Bobis, Clarke, Clarke, Gould, Thomas, Wright, & Young-Loveridge, 2005).

I outline below examples from two projects and the experiences of the authors in developing, piloting, and using interviews within professional development contexts. The potential of such interviews for enhancing teacher content knowledge and knowledge for teaching (Hill & Ball, 2004) is discussed. It will be argued that the use of such interviews can enhance many aspects of teacher knowledge, with consequent benefits to students.

The Early Numeracy Research Project

The Early Numeracy Research Project (ENRP) research and professional development program conducted in Victoria from 1999 to 2001 in Years Prep to 2 (with some limited data collection of the original Prep cohort in Years 3 and 4, in 2002 and 2003 respectively), investigated effective approaches to the teaching of mathematics in the first three years of schooling, and involved teachers and children in 35 project ("trial") schools and 35 control ("reference") schools (Clarke, 2001; Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). In all, the project involved 353 teachers and over 11 000 students of ages 4 to 8.

There were three key components to this research and professional development project:

- the development of a research-based framework of "growth points" in young children's mathematical learning (in Number, Measurement and Geometry);
- the development of a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively); and
- extensive professional development at central, regional and school levels, for teachers, coordinators, and principals.

As part of the ENRP, it was decided to create a framework of key "growth points" in numeracy learning. Students' movement through these growth points in trial schools, as revealed in interview data, could then be compared to that of students in the reference schools. The project team studied available research on key "stages" or "levels" in young children's mathematics learning (e.g., Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; Mulligan & Mitchelmore, 1996; Owens & Gould, 1999; Wilson & Osborne, 1992; Wright, 1998; Young-Loveridge, 1997), as well as frameworks developed by other authors and groups to describe learning.

The decision was taken to focus upon the strands of Number (incorporating the domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies), Measurement (incorporating the domains of Length, Mass and Time), and Geometry (incorporating the domains of Properties of shape, and Visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There were typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children "count-all" to find the total ("1, 2, 3, ..., 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children's developing understanding of Addition.

These growth points informed the creation of interview tasks, and the recording, scoring and subsequent data analysis, although the process of development of interview and growth points was very much a cyclical one. In discussions with teachers, I have come to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of mapping of the conceptual landscape. However, I do not claim that all growth points are passed by every student along the way.

The one-to-one interview was used with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 50-minute period, depending upon the interviewer's experience and the responses of the child. The interviews were conducted by the classroom teacher in trial schools, and a team of interviewers in reference schools. A range of procedures was developed to maximise consistency in the way in which the interview was administered across the 70 schools.

Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child moved through all of these. The interviewer made a decision after each task. Given success, the interviewer continued with the next task in the domain as far as the child could go with success. Given difficulty with the task, the interviewer either abandoned that section of the interview and moved on to the next domain or moved into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

The interview provided information about growth points achieved by a child in each of the nine domains. Below are two questions from the interview. These questions focus on identifying the mental strategies for subtraction that the child draws upon. The strategies used were recorded on the interview record sheet.

> 19) Counting Back
> For this question you need to listen to a story.
> a) Imagine you have 8 little biscuits in your play lunch and you eat 3.
> How many do you have left? ... How did you work that out?
> *If incorrect answer, ask part (b):*
>
> b) Could you use your fingers to help you to work it out? *(It's fine to repeat the question, but no further prompts please).*
>
> 20) Counting Down To / Counting Up From
> I have 12 strawberries and I eat 9. How many are left? ... Please explain.

It was intended that the interview would provide a challenge for all children. Over 36,000 interviews were conducted by teachers and the research team during the ENRP, and only one child was successful on every task — a Grade 2 boy in the second year of the project. It appeared that the aim of challenging all was achieved, with one possible exception!

Australian Catholic University Rational Number Interview

Following the perceived success of the Early Numeracy Research Project, it was decided to develop a one-to-one interview for teachers of nine- to fourteen-year olds. Given the recognised difficulty with fractions and decimals for many teachers and students (see, e.g., Behr, Lesh, Post, & Silver, 1983; Kieren, 1988; Lamon, 1999; Steinle & Stacey, 2003), it was decided to make rational numbers the focus of the interview. Anne Roche adapted and developed tasks in decimals (see, e.g., Roche, 2005; Roche & Clarke, 2004) and Annie Mitchell in fractions (see Mitchell & Clarke, 2004; Mitchell, 2005). In 2005, Clarke, Roche and Mitchell collaborated with Jan Stone (Association of Independent Schools, New South Wales) and Professor Richard Evans (Plymouth State University) in refining these tasks. A major source of tasks included the Rational Number Project (Behr & Post, 1992).

Once again, the selection of tasks used by the teacher is made during the interview, according to students' responses. There are currently 31 tasks assessing fraction understanding, 14 assessing decimal understanding, and 3 assessing proportional reasoning. Development on a range of tasks for percentages is continuing. To this point, approximately 70 teachers have been involved in piloting the tasks with their students. Two sample tasks are given in Figure 1.

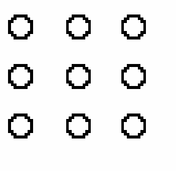
Nine dots	**Ordering**
Show the student the picture of 9 dots.  If this is three-quarters of a set of dots, how many dots is two-thirds of the set? *(drawing is okay if necessary) ……….* Please explain your thinking. [adapted from Cramer & Lesh, 1988]	*Place the cards randomly on the table.* Put these numbers in order from smallest to largest. *Encourage the student to think out loud while ordering them* **a) 0 0.01 0.10 .356 0.9 1 1.2** *Show each card below in turn* **1.70 1.05 .10** b) Where would this decimal go? Why does it belong there?

Figure 1. Sample tasks from the Australian Catholic University Rational Number Interview.

It should be noted that the task of developing "growth points" or a learning and assessment framework in rational number understanding is proving more elusive than for the domains of the ENRP. At present, our compromise is a statement of 25 "big ideas" in rational number knowledge, skills and understanding.

For example, one big idea is “works within a variety of physical and mental models (areas and regions, sets, number lines, ratio tables, etc.), in continuous and discrete situations.” However, because the domain of rational numbers is made up of many aspects or “subconstructs” (Kieren, 1988), and the use of many models within each subconstruct (Lamon, 1999), it has been a challenging task to try to map out a “conceptual landscape” for this content.

Similarly, it has been difficult to arrange the interview tasks in the same way as the ENRP, with many “drop-out points” and detours, as I have found that success or lack of success on a given task is not necessarily a good predictor for performance on another task, even when they seem closely related.

In the following sections, particular tasks and insights from teachers will be used to build the argument of the power of the interview as a professional development tool. I will outline the benefits to teacher professional growth and therefore the quality of teaching of the use of task-based, one-to-one interviews by mathematics teachers in the early and middle years of schooling.

INTERVIEWS AS A POWERFUL TOOL FOR MATHEMATICS TEACHERS

In the remainder of the paper, I will use data collected from teacher surveys as supporting data, and anecdotes from our own experience, a combined total of approximately 500 interviews.

Higher quality assessment information

In contrast to the traditional pen and paper test, a carefully-constructed and piloted one-to-one interview can provide greater insights into what students know and can do. Student strategies are recorded in detail on the interview record sheet. For example, in addition and subtraction, for the two subtraction tasks outlined earlier in this article, the teacher completes the record sheet, as shown in Figure 2, recording both the answer given and the strategies used. The emphasis on recording both answer and strategies is clear recognition that the answer alone is not sufficient.

The act of completing the record sheet requires an understanding of the strategies listed (e.g., modelling all, fact family, count up from, etc.). The use of the interview is therefore building pedagogical content knowledge (Shulman, 1987).

The capacity of the teacher to take the information on the record sheet and “map” student performance in relation to the growth points or “big ideas” is a key step in the process. Teachers after conducting the interview are likely to ask the reasonable question in relation to planning, “So now what?” If they have a clear picture of individual and group performance in particular mathematical domains, they are then in a position, hopefully with support of colleagues, to plan appropriate classroom experiences for individuals and groups.

19. Count Back / Modelling All (8 – 3)

a. Answer .... ❐

- ***Known fact or fact family (eg., 5 + 3 = 8)***
- ***Count back all, in head (7, 6, 5 or 8, 7, 6, 5)***
- ***Count back all, with fingers used to keep track only (7, 6, 5 or 8, 7, 6, 5)***
- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

b. Answer .... ❐

- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

20. Count Down To / Count Up From (12 – 9)

Answer .... ❐

- ***Known fact or fact family (eg., 9 + 3 = 12)***
- ***Count down to (12, 11, 10, 9)***
- ***Count up from (9, 10, 11, 12)***
- ***Fingers used during "count down to" or "count up from" to keep track only***
- Count back all (12, 11, 10, 9, 8, 7, 6, 5, 4, 3)
- Modelling all (shows 12 "things", then takes away 9 "things", leaving 3)
- Other ....

Figure 2. An excerpt from the addition and subtraction interview record sheet.

A focus on mental computation

Northcote and McIntosh (1999), using surveys of all reported computations of 200 adults over a 24-hour period, concluded that approximately 83% of all computations involved mental methods, with only 11% involving written methods. In addition, they found that over 60% of all computations only involved an estimate. These findings influenced greatly the construction of our interviews, where mental computation and estimation feature prominently.

Physical involvement: Making the task match the desired skill

Some mathematical skills and understandings can be very difficult to assess without some kind of physical task. As one teacher wrote, "to see whether children can do physical things, we sometimes need to watch." Consider this task from the Place Value section of the ENRP interview. The child is given a pile of icy-pole sticks, 7 bundles of 10 sticks each wrapped in an elastic band and about 20 loose ones. The teacher explains to the child that there are "bundles of ten and some more loose ones." The child is then shown a card with the number "36" on it, and asked to "get this many icy-pole sticks."

The student response is a very helpful indicator of place value understanding, in that some will feel the need to pull apart the bundles of ten (possibly indicative of an understanding of 36 only as the number in the sequence 1, 2, …, 36); some will count "10, 20, 30," and then "1, 2, 3, 4, 5, 6." A subtle improvement is the child who is able to say "3 of these and 6 of those," without any need to count. It is difficult to imagine a task that didn't involve this level of physical action providing the same opportunity for the teacher to gain what they do from listening and watching.

Objects of various kinds also increase the level of accessibility to tasks, and enjoyment of the experience for the student. There is also a number of topics in the mathematics curriculum which are not easily assessed by traditional means, e.g., visualisation and orientation, and manipulation of objects allowed students to show what they know.

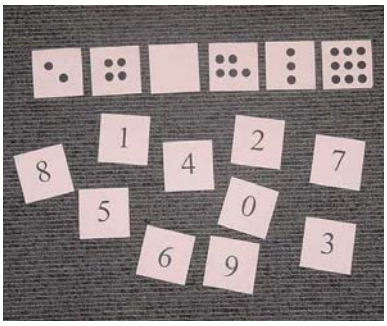
Large scale valid and reliable data

Processes used by the research team to maximise reliability and validity of interview data have been detailed elsewhere (see Clarke, 2001, Clarke et al., 2002). Having data on over 36 000 ENRP interviews across Grades Prep to 4 (the project focused on Grades Prep to 2, but a small "spin-off" project involved interviews with over 1000 students at each of Grades 3 and 4), provided previously-unavailable high quality data on student performance. These data had several benefits:

- Information for teachers on what "typical performance" for various grade levels looked like enabled them to relate the performance of their students to that of the cohort. For example, Table 1 shows the percentage of children on arrival at schools in trial schools who were able to match numerals to their corresponding number of dots. The data on children in the first year of school is discussed in considerable detail in Clarke, Clarke and Cheeseman (2006).

  Table 1. Performance of trial school children on entry to school in February 2001 on selected tasks (%) (*n* = 1437)

	Percent Success
Match numeral to 2 dots	86%
Match numeral to 4 dots	77%
Match numeral to 0 dots	63%
Match numeral to 5 dots	67%
Match numeral to 3 dots	79%
Match numeral to 9 dots	41%

  Teachers and researchers found considerable variation within classes in what students knew and could do, to an even greater extent than many previously thought. Of course, this makes a mockery of arguments that "all Prep children should be studying this and not that."

- Information is available for state departments of education and curriculum developers to inform their work. One of the most powerful pieces of data which is hopefully informing the development of the Victorian Essential Learning Standards (Victorian Curriculum and Assessment Authority, 2005) is found in the domain of Addition and Subtraction. Achievement of the growth point "Derived strategies in addition and subtraction" was assessed by the following tasks:

12 - 6   7 + 8   19 - 15   16 + 5   36 + 9

- Students were deemed to have achieved the growth point if they answered correctly (mentally, with no time limit), and used at least three preferred strategies across the five problems. For example, for 36 + 9, counting by ones ("36, 37, 38, …, 45") is a non-preferred strategy, while 36 + 10 - 1 would be a preferred strategy.

  At the end of Grade 2, only 19% of "typical children" could succeed on this basis. Even in trial schools (where teachers had been given intensive professional development), the percentage was only 31%. Yet, at the time, the state curriculum guidelines implied that virtually all children should be able to do these tasks. In light of these data (and the figure for typical students at the end of Grade 4—55%), it would appear that the state curriculum needs revision in terms of this content, as well as a consideration of whether the common practice of introducing conventional algorithms as early as Grade 2 is completely inappropriate (see Clarke, 2005 for more on this issue).

Building a knowledge of variations in performance across grade levels

It is interesting to collect sufficient data in order to observe trends in development of student understanding across the grade levels. To illustrate this point, a task adapted from the Rational Number Project (Cramer, Behr, Post & Lesh, 1997; Cramer & Lesh, 1988) and used in our Rational Number Interview, is shown in Figure 3.

Figure 3. A task used in the Australian Catholic University Rational Number Interview.

Fraction pie

Show the student the pie diagram.

a) What fraction of the circle is part B?.....
How do you know that?
b) What fraction of the circle is part D?.....
How do you know that?

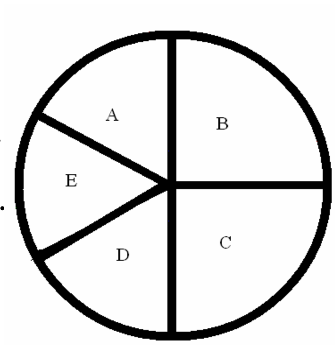


Table 2 shows student performance by grade level on the two parts of this task. To be correct, both the correct answer and an appropriate explanation were required. Students who were unsuccessful on part (a) were not given part (b) to attempt. Once again, the difficulty posed by this task for many students, possibly due to a lack of familiarity with tasks where not all parts are the same size, has implications for both emphasis and the pace of moving through content in fractions.

Table 2. Student Performance on Part-Whole Task (Continuous Case) by Grade Level (Years 4-6)

Q4 Part B Pie			
Grade	4	5	6
Correct	35/58	52/68	50/61
%	60%	76%	82%

Q4 Part D Pie			
Grade	4	5	6
Correct	29/58	36/68	33/61
%	50%	53%	54%

One of the advantages of administering the assessment interview at both the beginning and end of the school year was that teachers were provided, face-to-face, with exciting evidence of growth in student understanding over time.

Relating performance in one part of the interview to performance in another part

It is informative for teachers and researchers to consider whether understandings evident in one part of the interview prove accessible in another context. A major feature of teaching for relational understanding (Skemp, 1976) is that understanding enhances transfer (Hiebert & Carpenter, 1992). Among the possible tasks a student might encounter in the ENRP interview Counting section were tasks asking them to count by 2s, 5s, and 10s from 0. Given success, they counted by 10s and 5s, from 23 and 24 respectively.

In the Multiplication and division part of the interview, as part of a task assessing what we called abstracting multiplication and division (see Sullivan, Clarke, Cheeseman, & Mulligan, 2001), students were shown an array of dots which was then partially-covered as shown in Figure 4. They were then asked: "How many dots altogether on the card?" Even when students who counted by ones were prompted by, "could you do it a different way, without counting them by ones," the success rate was not high. Only 37.5% of 2942 Year Prep to 2 students were successful in transferring those skills to this new context.

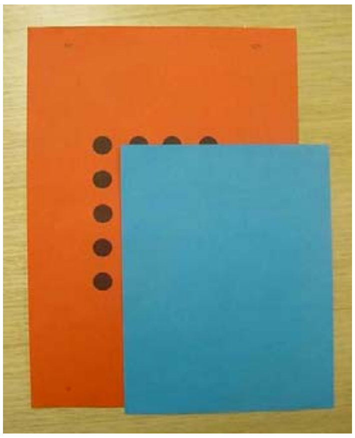


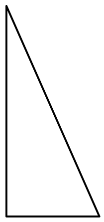
Figure 4. An array task.

Enhanced teacher knowledge of mathematics

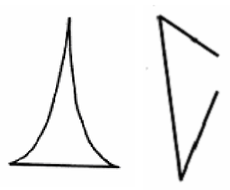
Our experience in working with teachers is that the use of the interviews enhances teacher content knowledge. In the middle years, many teachers acknowledge their lack of a connected understanding of rational number, often using limited subconstructs (sometimes only part-whole), and limited models (such as the ubiquitous "pie"). Many teachers have reported that their own understanding of rational number (e.g., an awareness of subconstructs of rational number such as measure and division and the distinction between discrete and continuous models) has been enhanced as they observe the variety of strategies their students draw upon in working on the various tasks and complete the record sheet.

Some might presume that teacher content knowledge is not an issue. However, many teachers reported that terms such as "counting on," "near doubles", and "dynamic imagery" were unfamiliar to them, prior to their involvement in the ENRP. It is interesting to consider whether this is content knowledge or "knowledge for teaching" (see, e.g., Ball & Bass, 2000; Ball & Hill, 2002; Hill & Ball, 2004).

Teachers develop an awareness of the common misconceptions and strategies which they may not currently possess



As teachers have the opportunity to observe and listen to students' responses, they become aware of common difficulties and misconceptions. For example, many children in Years Prep to 4 were unable to give a name to the shape on the left. It wasn't expected that they would name it "right-angled triangle," but simply "triangle". Because it didn't correspond to many students' "prototypical view" (Lehrer & Chazan, 1998) of what a triangle was (*a triangle has a horizontal base and "looks like the roof of a house"—either an isosceles or equilateral triangle*), some called it a "half-triangle, because if you put two of them together you get a real triangle." Many students nominated the two shapes on the right as triangles.



It was clear from a teaching perspective that it was important to focus on the properties of shapes, and to present students with both examples and non-examples of shapes.

The quiet achievers sometimes emerge

> *In every class there is that quiet child you feel that you never really 'know'—the one that some days you're never really sure that you have spoken to. To interact one-to-one and really 'talk' to them showed great insight into what kind of child they are and how they think (ENRP teacher, March 1999, quoted in Clarke, 2001).*

In response to a written question on highlights and surprises from the Early Numeracy Interview, a number of teachers noted that the one-to-one interview enabled some "quiet achievers" to emerge, and several noted that many were girls. There appeared to be some children who didn't involve themselves publicly in debate and discussion during whole-class or small-group work, but given the time one-to-one with an interested adult, really showed what they knew and could do.

> *The greatest highlight was that no matter at what level the children were operating mathematically, all children displayed a huge amount of confidence in what they were doing. They absolutely relished the individual time they had with you; the personal feel, and the chance to have you to themselves. They loved to show what they can do (ENRP teacher, March 1999, quoted in Clarke, 2001).*

Improved teacher questioning techniques (including the use of wait time)

Teachers noted that the interview provided a model for classroom questioning, and as a result of extensive use of the interview, they found themselves making increasing use of questions of the following kind:

- Is there a quicker way to do that?
- How are these two problems the same and how are they different?

- Would that method always work? . . .
- Is there a pattern in your results?

Teachers also observed the power of *waiting* for children's responses during the interview, noting on many occasions the way in which children who initially appeared to have no idea of a solution or strategy, thought long and hard and then provided a very rich response. Such insights then transferred to classroom situations, with teachers claiming that they were working on allowing greater wait time.

Tasks provide a model for classroom activities

Teachers were strongly discouraged from "teaching to the test" through presenting identical tasks to those in the interview during class. Nevertheless, the tasks did provide a model for the development of different but related classroom activities. For example, in the Place Value section of the Early Numeracy Interview, students are asked to type numbers on the calculator as they are read by the teacher or read numbers that emerge as they randomly pick digits and extend the number of places (ones, tens, hundreds, etc.) of the number on the screen.

Seeing the potential of the calculator as a tool for exploring and extending place value understanding, teachers would try tasks such as "type the largest number on the calculator which you can read (but no zeros in it)." The reason for the instruction to have no zeros in the number was because some children will be able to read a million, but not necessarily 386. Such a task provides an opportunity for the teacher to challenge them to make the number even larger. This task, re-visited regularly, provides a helpful measure of growth in student understanding over time, and therefore can be used as an ongoing assessment tool.

Teacher professional growth: Some final comments

At a professional development day involving all 250 or so teachers) towards the end of 1999, ENRP teachers were asked to identify changes in their teaching practice (if any), as a result of their involvement in the project. There were several common themes, many of which can be related to the professional growth experienced through the use of the interview:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on "pulling it together" at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and "real life mathematics".
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Several of the themes discussed in this article are evident in the following quote from a teacher who attended the professional development program:

> *The assessment interview has given focus to my teaching. Constantly at the back of my mind I have the growth points there and I have a clear idea of where I'm heading and can match activities to the needs of the children. But I also try to make it challenging enough to make them stretch.*

ONE –TO-ONE INTERVIEWS AND LESSON STUDY

So what is the potential relationship between the use of one-to-one assessment interviews and lesson study? In describing the Early Numeracy Research Project, we have sometimes used these words: "understanding, assessing and developing young children's mathematical thinking."

The growth points provide a way of *understanding* students' thinking and possible pathways or trajectories through which students might move, the interview provides a way of establishing where students "are at" in relation to these pathways (*assessing*), and the professional development program provided an opportunity to explore how this understanding might be developed further ("*developing*"). I would argue that lesson study fits very nicely in with the third aspect. If teachers have a clear picture of their students' understanding of mathematics and a framework against which this can be mapped, then lesson study provides an ideal model for planning "where to from here?" In this way, the use of one-to-one assessment interviews is in complete harmony with the lesson study approach.

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IMPLEMENTING LESSON STUDY
IN NORTH AMERICAN SCHOOLS AND SCHOOL DISTRICTS

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Because no professional development practice similar to lesson study exists in North America, it is often challenging for North American teachers and schools to implement lesson study. Lesson study has, however, become highly visible in many state, national, and international conferences, open houses, high-profile policy reports, and special journal issues in North America. Moreover, numerous schools and school districts in the United States have attempted to use it to change their practices and to impact student learning. This paper is intended to provide some ideas about how to conduct lesson study for the educators who are interested in implementing lesson study in their schools and school districts.

JAPANESE LESSON STUDY MODEL

The practice of lesson study originated in Japan. Widely viewed in Japan as the foremost professional development program for teachers, lesson study is credited with dramatic success in improving classroom practices in the Japanese elementary school system (Fernandez, Chokshi, Cannon, & Yoshida, 2001; Lewis, 2000; Lewis & Tsuchida, 1998; Shimahara, 1999; Stigler & Hiebert, 1999; A. Takahashi, 2000; Yoshida, 1999).

A particularly noticeable accomplishment in the past 20 years of lesson study in Japan has been the transformation from teacher-directed instruction to student-centered instruction in mathematics and science (Lewis & Tsuchida, 1998; Takahashi, 2000; Yoshida, 1999). The success of lesson study can be found in two primary aspects: improvements in teacher practice and the promotion of collaboration among teachers.

First, lesson study embodies many features that researchers have noted are effective in changing teacher practice, such as using concrete practical materials to focus on meaningful problems, taking explicit account of the contexts of teaching and the experiences of teachers, and providing on-site teacher support within a collegial network. It also avoids many features noted as shortcomings of typical professional development, e.g., that it is short-term, fragmented, and externally administered (Firestone, 1996; Huberman & Guskey, 1994; Little, 1993; Miller & Lord, 1994; Pennel & Firestone, 1996). In other words, lesson study provides Japanese teachers with opportunities to make sense of educational ideas within their practice, to change their perspectives about teaching and learning, and to learn to see their practice from children's perspectives. For example, a Japanese teacher said, "It is hard to incorporate new instructional ideas and materials in classrooms unless we see how they actually look. In lesson study, we see what goes on in the lesson more objectively, and that helps us understand the important ideas without being overly concerned about other issues in our own classrooms" (Murata & Takahashi, 2002).

Second, lesson study promotes and maintains collaborative work among teachers while giving them systematic intervention and support. During lesson study, teachers collaborate to: 1) formulate long-term goals for student learning and development; 2) plan and conduct lessons based on research and observation in order to apply these long-terms goals to actual classroom practices for particular academic contents; 3) carefully observe the level of students' learning, their engagement, and their behaviors during the lesson; and 4) hold post-lesson discussions with their collaborative groups to discuss and revise the lesson accordingly (Lewis, 2002). One of the key components in these collaborative efforts is "the research lesson," in which, typically, a group of instructors prepares a single lesson, which is then observed in the classroom by the lesson study group and other practitioners, and afterwards analyzed during the group's post-lesson discussion. Through the research lesson, teachers become more observant and attentive to the process by which lessons unfold in their class, and they gather data from the actual teaching based on the lesson plan that the lesson study group has prepared. The research lesson is followed by further collaboration in the post-lesson discussion, in which teachers review the data together in order to: 1) make sense of educational ideas within their practice; 2) challenge their individual and shared perspectives about teaching and learning; 3) learn to see their practice from the student's perspective; and 4) enjoy collaborative support among colleagues (Akihiko Takahashi & Yoshida, 2004).

Lynn Liptak, a principal who is pioneering lesson study in the U.S., argues that because lesson study is a teacher-led approach to professional development, teachers can be actively involved in the process of instructional change, in contrast with traditional professional development methods.

Contrasting methods of professional development

Traditional	Lesson Study
Begins with answer	Begins with question
Driven by outside "expert"	Driven by participants
Communication flow: trainer to teachers	Communication flow: among teachers
Hierarchical relations between trainer & learners	Reciprocal relations among learners
Research informs practice	Practice is research

(Reprinted from Lewis, 2002, p.12)

Lesson study also has played an important role in improving curricula, textbooks, and teaching and learning materials in Japan. In fact, most Japanese mathematics textbook publishers employ as authors classroom teachers who are deeply involved in lesson study, and their materials are in some manner examined through the process of lesson study.

The process of lesson study

Lesson study does not follow a uniform system in Japan. It is more like a cultural activity. As a result, lesson study takes many different forms, including school-based lesson study, district-wide lesson study, and cross-district lesson study. Therefore, there are neither clear definitions nor specified criteria of lesson study in Japan. Its process differs across schools, districts and types of lesson study. Lesson study groups can be formed by all the members in a school building, or by study-group members in a district, or by teachers who are interested in specific subject matter.

Although the types of groups differ, the lesson study process usually begins with identifying a long-term goal or goals or a research question or set of questions as a theme. Since lesson study is a way to bring educational goals and standards to life in the classroom (Lewis, 2002), this process usually involves all the members of the lesson study group. After a lesson study group establishes a theme, the cycles lesson study begin. A typical lesson study group activity involves several lesson study cycles in a year. A lesson study group usually divides into two or more sub groups each containing four to six teachers sharing a particular interest or teaching the same or similar grade levels. One of the sub groups, called the "lesson planning team," develops a lesson plan and conducts a research lesson. The other sub-group members who are not involved in planning the lesson but who observe the lesson, are called "research lesson participants." In each lesson study cycle, a different sub-group becomes the lesson planning team. A lesson study group sometimes invites teachers and university professors from outside the group as lesson study participants. Both lesson planning team members and lesson study participants play important roles and contribute differently to the lesson study project.

A major role of the lesson planning team is to develop a lesson plan. Based on this lesson plan, one of the teachers from the team teaches his or her class. This lesson is called the "research lesson" (*Kenkyuu-jugyou*) and is observed by all the members of the lesson study group. To develop a lesson plan, the group usually meets three to five times for sessions of two to three hours. The team members also prepare teaching and learning materials such as manipulatives and student worksheets for the lesson.

Following the research lesson, the lesson planning team and all the research lesson participants discuss whether the students in the class accomplish the goal or goals of the lesson. This is called post-lesson discussion (*Kenkyu-kyogikai*). A major role of the research lesson participants is to study the impact of the lesson in order to improve the lesson plan. To do this they need to collect data during the research lesson to support their arguments. Participants might collect various types of data, such as how many students actually solved the problem and how many different solution methods were discussed in the class, how particular students solved the problem during the lesson and how the class discussion helped these students to improve their solution methods, or how particular students summarized the class discussion in their notes. Participants may collect data differently depending on their interests and experiences. They may also interpret the data differently. As a result, a wide variety of data can be expected for a post-lesson discussion and will contribute to the richness of the post-lesson discussion and greatly help to improve the lesson plan. In this way each research lesson participant is expected to be like a researcher who collects data to examine whether the lesson plan facilitates student learning and whether the lesson plan need to be improved. The lesson planning team also plays an important role during the post-lesson discussion. They are expected to explain the discussion and rationale behind the lesson plan. This information helps participants better understand the lesson.

The activities of lesson study -- reading a lesson plan, observing a class, and examining the class in terms of student learning -- all benefit the research lesson participants in their larger roles as classroom teachers when they develop their own lesson plans and work to improve their own instruction.

Outside specialists (*Koshi*), so-called knowledgeable others, may also play an important role in lesson study. The knowledgeable other is typically invited as an advisor for the lesson planning team and as an outside commentator who summarizes the post-lesson discussion. Some schools and school districts engage the same knowledgeable others to continuously support their lesson study over a number of years. A lesson study group usually invites a person who has experience in the process of lesson study, and both pedagogical and content expertise, such as an experienced teacher, a university professor, or a district specialist. A knowledgeable other is expected not only to summarize the participants' discussion about the research lesson and draw out its important implications (Watanabe, 2002) but also to bring new perspectives to the lesson study group.

Throughout the lesson study process, teachers have opportunities to clarify how to apply particular educational ideas in their practice, to refine their perspectives on teaching and learning, to view their practices from the students' perspective, and to enjoy the collaborative support of their colleagues.

LESSON STUDY IN NORTH AMERICA

Many U.S. educators have recently become interested in lesson study as a promising source of ideas for improving education (Stigler & Hiebert, 1999). Within the last several years lesson study has become highly visible in many state, national, and international conferences, open-houses, high-profile policy reports, and special journal issues in North America. Moreover, some school districts in the United States have attempted to use it to change their practices and to impact student learning (Council for Basic Education, 2000; Germain-McCarthy, 2001; Research for Better Schools Currents Newsletter, 2000; Stepanek, 2001; Weeks, 2001).

As of September 2, 2003 at least 29 states, 140 lesson study clusters/groups, 245 schools, 80 school districts, and 1100 teachers across the United States were involved in lesson study. (Lesson Study Research Group). The following map, figure 1, shows some lesson study groups in North America.

Figure 1

Chicago Lesson Study Group

One of many lesson study groups in North America, the Chicago Lesson Study Group has become well known among lesson study researchers and practitioners as one of the few groups that conduct public research lessons.

To explore the possibilities for replicating the success of Japanese lesson study in a U.S. setting, the Chicago lesson study group was launched in November of 2002, with volunteer school administrators and classroom teachers who have had university student teachers in their classrooms as a part of their field experiences. About twenty members are active and another thirty follow the group's activities on an email list Although the most popular form of lesson study in Japan takes place within a single school as a school-based professional development program (Yoshida, 1999), the Chicago Lesson Study Group adopted a cross-school volunteer model for its lesson study group. The reasons for this adaptation are twofold. First of all, an effective model of lesson study is often one that is started as a grassroots movement of enthusiastic teachers rather than as a top-down formation (Lewis 2002; Takahashi & Yoshida, 2004; Yoshida, 1999). For this reason, starting a lesson study group as a cross-school volunteer group was thought to be appropriate. Furthermore, it is sometimes difficult to establish a school-based lesson-study group in the U.S. because many teachers do not have experience working with other teachers in the same school as a group to accomplish a shared goal. Secondly, in order to have a sufficient number of enthusiastic elementary and middle school teachers who are interested in lesson study focusing on mathematics, a cross-school model was found to be more appropriate in the U.S. setting.

The program of activities for a volunteer lesson study group usually consists of two components: (1) a series of study groups concerned with improving the teaching and learning of mathematics (the group usually meets after school regularly throughout the year), and (2) several public research lesson opportunities each year to examine the work of the study group by inviting a wide variety of individuals to participate in its sessions. Since its inception, this study group has met twice a month to discuss ways to implement the ideas of reform mathematics in order to improve the teaching and learning of mathematics.

In order to find a way to implement the ideas of reform mathematics, the Chicago lesson study group has conducted five lesson-study conferences with ten public research lessons in the past four years. In each conference, the group has invited teachers and educators from not only the Chicago area but also from other states to discuss how to implement student-centered classrooms in mathematics. About one hundred participants from various U.S. states and Canada have attended the conferences each year and discussed how to help students develop algebraic thinking skills through problem solving.

HOW CAN WE BEGIN LESSON STUDY?

Because no professional development program similar to lesson study exists in the North America, it is often challenging for North American teachers and schools to implement lesson study. In order for teachers and schools to overcome the hesitation to become a part of lesson study, the following suggestions are usually given to the North American teachers and schools who are interested in exploring the possibility for implementing lesson study.

Begin with an informal study group

Since lesson study is a form of teacher-led professional development, any teacher can begin lesson study by connecting with another teacher. This means that lesson study is a grassroots movement among teachers rather than a top-down formation. Forming informal study groups focused on improving mathematics teaching and learning can be a step toward developing a lesson study group. If you are not already part of such group, you might share what happened in your math class during a grade level meeting. You do not have to begin lesson study with all the teachers in a school building at once. Forming a comfortable collaborative group is the most desirable step toward developing a lesson study.

Experience lesson study

The idea of lesson study is simple: collaborating with fellow teachers to plan, observe and reflect on lessons. Developing a lesson study, however, is a more complex process (Lewis, 2002). Because lesson study is a cultural activity, an ideal way to learn about lesson study is to experience it as a research lesson participant. In so doing, you will learn such things as how a lesson plan for lesson study is different from a lesson plan that you are familiar with, why such a detailed lesson plan is needed, what type of data experienced lesson study participants collect, and what issues are discussed during a post-lesson discussion. The following websites are excellent for exploration of the lesson study topics:

Chicago Lesson Study Group (http://www.lessonstudygroup.net)

Global Education Resources (http://www.globaledresources.com)

Lesson Study Group at Mills College (http://www.lessonresearch.net)

National Council of Teachers of Mathematics (http://nctm.org)

Identifying your research theme

Since lesson study is teacher-led professional development, the participants determine the group's theme or topic. For example, the Chicago Lesson Study Group chose measurement as their theme because measurement was the worst area of mathematics for their students as reflected in standardized test scores, and because measurement was the most difficult topic for them to teach. This theme emerged from a discussion about which topics teachers found difficult to teach.

Investigate a variety of materials to develop a lesson plan for a research lesson

Even though a group has identified its theme, it is still too early to develop a lesson plan. Some groundwork is needed. For example, if a group decides to explore how to teach measurement of the area of a rectangle for fourth grade students, the group needs to know how this topic relates to the other topics in the same grade, what prior knowledge students should have, and how this topic will help students learn mathematics in their future classes. Moreover, teachers need to know what kind of materials various textbooks use to teach this topic to students, and what research suggests (if anything) about various methods for teaching the topic. This investigation, called '*Kyouzai-kenkyuu*' in Japanese, means studying. Kyouzai-kenkyuu typically investigates the following areas:

- a variety of teaching and learning materials, such as curricula, textbooks, worksheets, and manipulatives
- a variety of teaching methods
- the process of student learning including students' typical misunderstandings and mistakes
- research related to the topic

Japanese teachers often begin *Kyozai-kenkyu* by comparing various teacher's guides published by textbook companies. Thus, U.S. teachers start using the English translation of the Japanese mathematics textbook series (Global Education Resources[1], 2006) and teaching guides for the Japanese Course of Study as resources to conduct *Kyozai-kenkyu*.

[1] http://www.globaledresources.com

Developing a lesson plan

There are many different types of lesson plans in Japan. Although no single universal form is available, any lesson plan is expected to provide enough information for lesson study participants to learn why the lesson-planning group decided to use a certain problem for the students, why the group chose a particular manipulative for the class, and why the group used particular wording for the key questions. To explain these rationales, a typical lesson plan includes the title of the lesson, the goal of the lesson, the relationship of the lesson to the standards or curriculum, the "about the lesson", the expected learning process, and the evaluation. Use of a simplified lesson plan might be a good idea for a novice lesson study group. One of the most difficult sections for teachers to develop in a lesson plan is the section describing the rationale of the lesson. Experienced participants often read this section very carefully because they believe that it is the essence of Kyozai-kenkyu. If this section cannot tell participants enough information about the lesson, the group's Kyozai-kenkyu might not be deep enough. Usually, the lesson plan rationale includes discussion of the following:

a. Concepts or skills that the students need to learn in the lesson or unit according to the standards and/or curriculum
b. Concepts or skills that the students have already learned
c. The major focus (theme) of this lesson or unit by comparing (a) and (b) (the objective of this lesson should be clearly stated)
d. The way to help students accomplish the above objective as a hypothesis for the research lesson

Lesson study groups might be able to test their draft lessons plan prior to the research lesson in another member's class as a pilot lesson. By using the data collected during the pilot lesson, the group might revise the lesson plan in preparing the research lesson.

The following shows a typical schedule for developing a lesson plan by a lesson planning team.

- The first meeting (five weeks before)
  Identifying the team's research goal/theme
  Deciding on a topic to investigate
- The second meeting (four weeks before)
  Investigate a variety of resources and teaching materials to develop a lesson plan (*Kyozai Kenkyu*)
- The third meeting (three weeks before)
  Developing a research lesson and writing the lesson plan for the lesson
- The fourth meeting (second weeks before)
  Completing the first draft of the lesson plan
- <Option: Teaching a class based on the first draft>
- The Fifth Meeting (a weeks before)

Completing the final draft and prepare for the lesson

Conduct a research lesson and a post-lesson discussion

Respecting the natural atmosphere of the class is always a priority during a research lesson, so ideally a research lesson should be held in the instructor's regular classroom. However, if the regular classroom cannot hold enough participants, a research lesson might be taught in a larger classroom. Further, out of respect for maintaining the natural environment, neither members of the lesson planning group nor participants should give any advice or comments to the students, because the instructor is the only person who can teach the students.

A post-lesson discussion is usually held right after the research lesson. It might be a good idea to have a post-lesson discussion in the classroom where the research lesson was held because participants can see all the blackboard writing and materials that the students used during the class. Customarily the post-lesson discussion session begins with an instructor's short comment on his or her teaching. An explanation of the lesson plan by a member of the lesson-planning group follows. Next, data collected by the participants may be discussed, followed by a more general discussion, which is sometimes focused on topics identified in advance. Although any critique and comments should be welcomed, a facilitator often keeps the discussion focused on the issues of interest to the planning group, rather than having a "free-for-all." At the end of the session, an outside specialist (Koshi) is given an opportunity to make a final comment as a summary of the session. The post-lesson discussion session should be recorded by a note taker. More guidelines for lesson observations and post-lesson discussions are available in *Lesson Study: A Handbook of Teacher-Led Instructional Change* (Lewis, 2002) and *Currents*, spring/ summer 2002 (Research for Better Schools, 2002).

LET'S BEGIN LESSON STUDY

Research suggests that mathematics class should be shifted from traditional teacher-led instruction to student-centered instruction. As a result, many schools and teachers are working hard to change their classrooms. However, most professional development programs are still done in a traditional way. The lesson study approach permits teachers involved in professional development to become as active in their learning as they expect their students to be.

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NESTING FEATURES OF DEVELOPING TEACHERS' PERSPECTIVES: A LESSON STUDY PROJECT FOR PROSPECTIVE TEACHERS IN MATHEMATICS WITH HISTORY AND TECHNOLOGY

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For teacher education with technology, we should consider many questions but there are no answers without focusing on the parameters. Here we discuss about a case for developing a good teacher's perspective through Lesson Study in terms of Japanese meaning with technology. Firstly, we define desirable teachers' perspectives. Secondly, we focus on the function of technology and history for teacher education. Thirdly, we analyze the case for explaining the developing process of teachers' perspectives in it.

I. Development of Teacher's Perspectives 'Kodomo wo miru me' for Mathematics

In the APEC meeting 'Innovative Teaching Mathematics through Lesson Study' in January 2006 at Tokyo, Catherine Lewis (2006) talked about her experience on Lesson Study in her keynote lecture as follows: (In her Lesson Study project) A U.S. teacher said as follows: "Before the Lesson Study, we had talked about multiple intelligence, constructivism and so on, but never talked about each subject matters of teaching. In the Lesson Study project, we began to talk about subject matters, why we teach them, how we teach them and what students learn from the lesson". In Tokyo's session, majority of participants may feel that this episode is not just for U.S. but for all countries. In the in-service teacher training programs, mathematics educators used to teach the theory of mathematics education. A comment from a teacher implicates that we teach theory and policy of curriculum and failed to teach them with subject matters. Multiple intelligence theory made us notice desirable competency which is not developed by one subject. In curriculum, teachers are expected to develop it through their lesson through teaching contents.

Constructivism theory promoted our awareness of the importance of listening students' ideas because students construct their knowledge by themselves. In teaching context, teacher's listening is not passive action such as only hearing but positive action (Arcavi & Isoda, to appear). Good lessons based on constructivism expect student-centralized lesson and the roles of teachers to conduct students' activity for their learning. In this context, listening activities by teachers are aimed to think about and find the way how to develop students' ideas to sophisticate or elaborate with others in their classrooms.

Lesson Study is an authentic activity for enabling teachers to conduct their classrooms. It includes discussions of subject matters, why they teach, how they teach and what students can learn.

Catherine also noticed her experience as follows; One teacher said, "I developed the eyes (teacher's perspective) to look at students and subject matters "Kodomo wo miru me". Now, I am well aware of my responsibility for my lesson. In the lesson study with other teachers, I preferred the more challenging lesson such as with Open-ended problems. When I found that students can challenge such difficult problems, I recognized self-confidence in my lessons". Catherine mentioned that teachers developed the ability to listen to students' ideas such as 'Kodomo wo miru me' but it is not only hearing (See such as Catherine

Lewis 2002). Because they developed good teachers' perspectives, they can say the development of eyes for understanding students and want to challenge the lesson with Open-ended problems and feel self-confidence through conducting the lesson.

Based on Japanese ideas of Lesson Study, teacher's perspective 'Kodomo wo miru me' is explained as the following (Isoda, Stephens, Ohara and Miyakawa, to appear): In Lesson Study, teachers discuss about the subject matter before the lesson. Teachers share responses (including misunderstandings) from students in the past lessons, have a lot of expectations about students' ideas and prepare their questions to extract students' ideas and their reaction against students' ideas. At the same time, teachers also expect that students' ideas will be more than their expectations. If students' ideas are within expectations, it is easily understandable for the teacher. Even if not, it is also within their expectations because it is a good chance for them knowing unknown ideas from students.

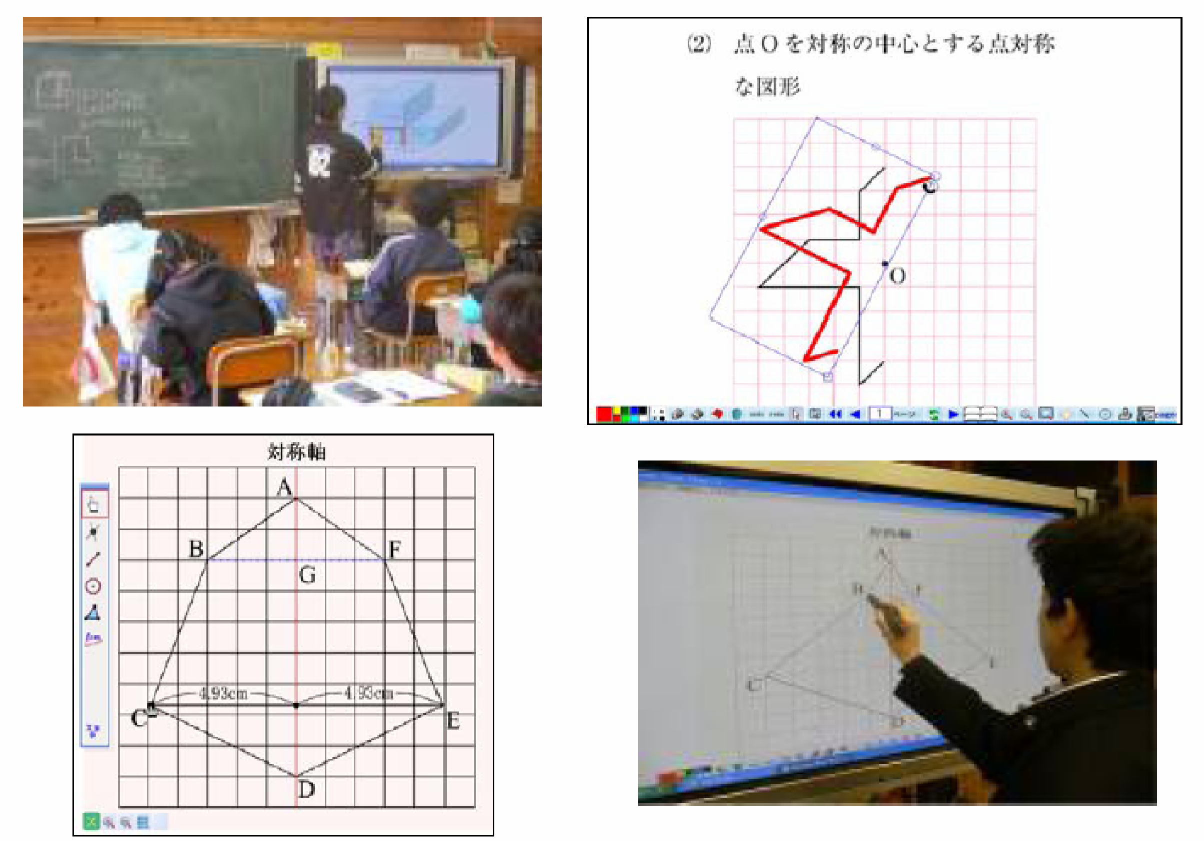
In teacher education, it is necessary to develop teachers for stepping up from listening to conducting. What necessary conditions for stepping up are and what kinds of processes are important for it even if there are no sufficient conditions. For example, some good teachers teach students the value what is important for life in any time and believe mathematics teaching is a part of the value education (Alan Bishop et al. 2003). Some novice teachers act differently between mathematics class and homeroom class. They worry how to solve and how to teach mathematics problems in every lesson but in homeroom activity, they try to push students' decision making. Through the experience, we can expect novice teachers to develop themselves to integrate their teaching contents and value. In this paper, the conditions and the processes are discussed as a case study.

II. Technology and History for Knowing Mathematics Differently

1. Minimum necessity to use technology for teacher education.

e-Larning is a current technology movement in education. Developing knowledge bank with learning management system is a trend. Equipment in schools and environment of internet are well known obstacles in general. But even if equipped, each teacher's belief of mathematics is an obstacle because mathematics is already embedded in physical or psychological tools such as papers, pencils and calculations. It is not easy to change teachers' beliefs because if we change tools then we have to change our mathematics itself. If we think their believes as an obstacle, we can not change. On contrary, If we recognize that each technological tool has it's own way of knowing mathematics differently, technology supports teacher educators to teach school mathematics differently and it may be a cue for next step.

For example, in mid of 90's, I engaged in in-service teacher training summer course to use a Graphing Calculator, Computer Algebra System and Dynamic Geometry Software during 5 years. The number of participants is more than fifty every year and half of them are repeaters. They enjoyed mathematics with technology, got the skill how to use and develop lesson plans for their classroom. But most of them did not use computers and graphing calculators in their classrooms because mathematics had been taught without technology and most subject matters in textbooks are not necessary to use technology. Between lines in textbooks, there are many things that should be taught. In a simple algebraic calculation from a line to a line, there are things which should be explained. Teacher can not alternate it to technology. In 90's, most of the technology developed as the environment and some mathematics educators believed to alternate the hidden aims in textbooks with technological environment. Indeed, we are now in a process of alternating textbooks to e-textbooks. The difference is that e-textbooks are a kind of textbooks. Teachers do not need to learn the commands how to use and integrate their aim of teaching with technological environment in the classroom (See Picture 1, Isoda et al. 2005).

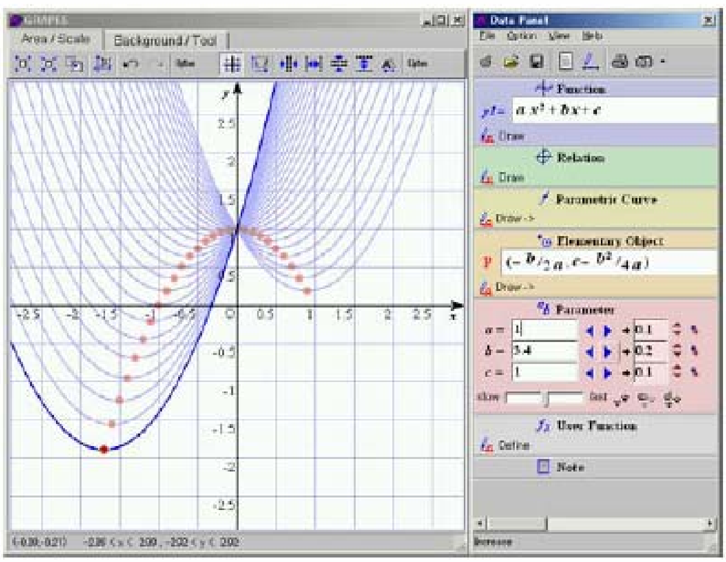


Picture 1. Using e-textbook with Interactive Board in classroom (Isoda et al. 2005)

What is obscure for me is that why many teachers had participated in summer courses even if they did not have a wish to alternate. I could say that they enjoyed knowing mathematics from different ways with technology. They enjoyed explorations of mathematics via technology. For example, if we draw graphs of $y = ax^2 + bx + c$ by fixing two parameters from a, b and c and changing one remained parameter regularly (Picture 2), we can find the role of each parameter, a, b or c which is never known by algebraic deduction to $y = (x-\alpha)^2 + \beta$.

Even if teachers did not have a chance to use computers or graphing calculators in their schools, exploring mathematics with technology in summer course is an enjoyable experience for them because it is the chance to know their known mathematics differently. If we use unknown technology, teachers can explore their school mathematics as unknown.

If we say that minimum necessity is needed to use technology in teacher education, we can say that it gives prospective or in-service teachers to explore school mathematics as a really new one. Teachers can re-experience their mathematics like students who learn from the beginning. Even if it is impossible because they already know, knowing differently is meaningful. If teachers know how to enjoy mathematics, it supports teachers enabling students to enjoy mathematics.



Picture 2. Grapes (Isoda et al. 2005)

2. Any Technology is innovative for knowing mathematics differently.

When we think about a function of technology in mathematics teacher education knowing mathematics differently, it is not necessary to focus on innovative technology because if we change technological or psychological tools (James Wertsch. 1991) we know mathematics differently. For example, if I have a card written with the number 2 in my left hand and I have a card with the number 6 in my right hand, and ask pupils to read cards, they must read two and six. If we bring closer both cards and ask the same question, what will happen? Pupils may begin to read twenty six. Even if we know that is the definition, we re-aware the difficulty and marvelous features of base ten system. Number Cards enable us to re-aware mathematics.

In elementary school mathematics, we usually use concrete materials for understanding. It is supported by not only Piaget' constructivism but also the theory of embodiment (George Lakoff, Raffael Nunez., 2000). For prospective teachers training, concrete materials are usually reused for teaching the methods of teaching because prospective teachers forgot how they learned content but prospective teachers enjoy like students before knowing it as the methods. For example, in picture 3 (MEXT, 2002), please find the price of an apple and the price of an orange posed with the picture without simultaneous equations. If you can solve it by operation of apples and oranges, you can enjoy unexpected explanation of the algebraic solution of simultaneous equations. Prospective teachers can recognize algebra as generalized operations of concrete objects.

Picture 3. How much each?

What implicates from these three examples here is that mathematical awareness is given with tools. Any technology for mathematics can be innovative for knowing mathematics differently.

3. Mathematics history as tools for cultural awareness

For knowing mathematics differently, mathematics itself can be useful. Indeed, mathematics is a psychological tool as for mediational means from the view point of Vygotskian theory (James Wertsch. 1991). History of mathematics itself is another mathematics when comparing with the current school mathematics. For mathematics teachers, I have been developing a web site in mathematics and history (See Picture 4. Isoda). It is not the web site of history itself. It's aim is to know mathematics differently and the origins from history. Most of contents are inspired

Picture 4. Mathematics History Museum by the Lesson Study Project (Isoda2005)

from historical texts in mathematics but with added educational view points. For example, in picture 4, it explains how to use sextant which was used for navigation before the age of radar and GPS. It tells us how high school mathematics was useful and necessary. A case study described in the next chapter is the Lesson Study Project that developed this website.

III. A Case Study of Developing Teachers' Perspective 'Kodomo wo miru me'

1. The introduction of Lesson Study in Japanese teacher education

It is difficult for prospective teachers to think like experienced teachers even if they take classes on a particular academic subject or on materials study. Thus, in teacher education programs in Japan, prospective teachers engage in micro-teaching exercises in which they engage in role playing, alternately playing the role of the teacher and the student to acquire the perspectives of both teacher and learner. They also participate in teaching internships of one month during which they do on-site training in an actual school. This allows students to become familiar with the cyclical Lesson Study process of researching materials, conducting Study Lessons, and holding feedback meetings to facilitate improvement. In the final week of their teaching internships, prospective teachers invite their advisors from the university to participate in their own Lesson Study project at the school.

2. A case study of Master Program in Education, University of Tsukuba

Becoming teachers by obtaining their Rank 1 Teaching Certificate in a master's degree program are trends in Japan. Each university's master's degree program offers its own excellent and distinctive teacher's education programs. Teacher education programs that cultivate the ability to lead practical and useful educational research are especially welcomed by teachers, the board of education, and the Ministry of Education, Culture, Sports, Science and Technology.

The Mathematics Course of the University of Tsukuba Master's Program in Education, which aims to train teachers for high school and beyond, addresses both pure mathematics and mathematics education. In the two year master program in education, we intend to develop leading teachers in mathematics education in school or university based on the tradition of ecole normale from 1873. Based on the image of leading teachers, following conditions are expected in this case study: 1) Good teachers can lead Lesson Study in their school, 2) Good teachers can teach other teachers how to use technology in mathematics from the beginning of his work, and 3) Good teachers can lead in the society of mathematics education.

In their first year of two year program, graduate students (prospective teachers) develop original mathematics teaching materials, conduct a three-hour Lesson Study project and write the research report for describing students' achievements. The project is done as a part of mathematics education class with six credits.

2-1. Aims and schedules on the Lesson Study project:

The Lesson Study project aimed to develop materials for giving high school students cultural awareness in mathematics, improve their attitudes and brief in mathematics by conducting lessons, and to demonstrate the educational value of the developed materials. The schedule to engage in the Lesson Study in the school year 2001 was as the following;

<u>Phase 1) Transition period</u> (almost April – June): Teacher educator (project director) explained first-year students a year plan of the project and explained what kinds of activities were expected. Second-year students in master program who engaged in last year's projects conduct new first-year students' classes to review the activities from their

actual lessons on the previous year's project. First-year students learned how to use the computers in their Lesson Study from second year students and began the project.

Phase 2) Reading of historical sources in mathematics (almost July – August): Students read historical textbooks (English readings or Japanese translations of primary sources) for excavating teaching materials and *A History in Mathematics Education* (John Fauvel, Jan Van Maanen. 2000) for learning the educational value and teaching methods of mathematics history. Teacher educator supported their reading, made clear interesting points when compared with today's mathematics and excluded the misinterpretation originated from reading mathematics history books with today's mathematics such as Bourbaki.

Phase 3) Subject matter development (almost September – November): Students developed subjects from historical texts, conceptualized lessons, established aims and goals, and developed teaching materials such as textbooks using original (or English translation) texts, slides and activities with computer. Teacher educator helped to find interesting materials from historical texts and supported students to develop structures of textbooks and lessons.

Phase 4) Lesson implementation (almost November – December): Students conducted the lesson. Teacher educator supported students to expect classroom students' activities, especially classroom students' responses and how teachers can use the response. Teacher educator also supported how to use classroom equipments such as projecting students' notebook activities to the screen for sharing students' ideas in the classroom.

Phase 5) Report preparation (almost December – February): Students wrote their research reports, created their web site. Teacher educator supported their references depending on their research problems and also supported their preparations for presentations among the mathematics education society.

IV. Analysis of the Case

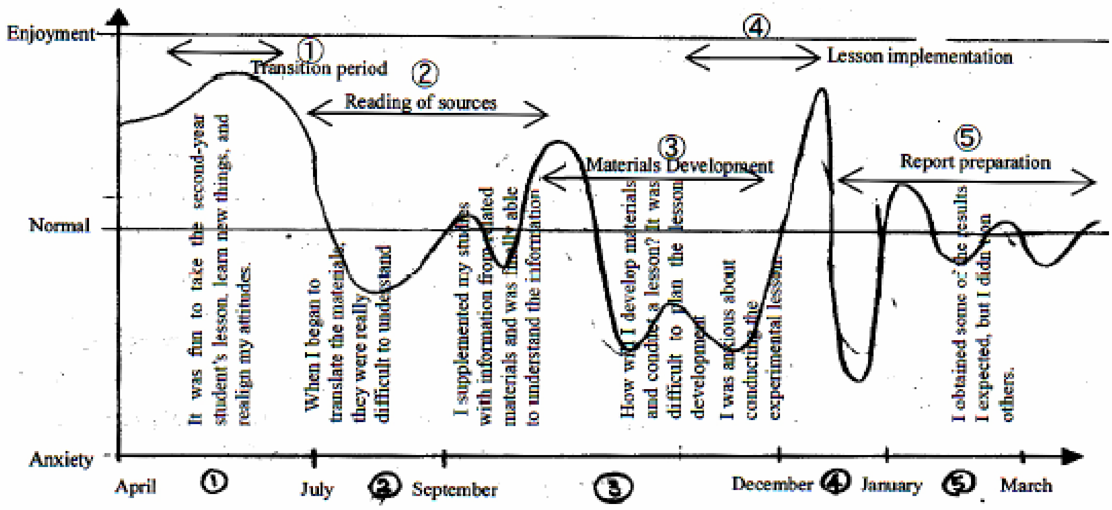
1. Analysis of the prospective teachers' experience through the project

Fourteen prospective teachers in master program participated in the project at school year 2001. After the phase 5, the researcher asked to represent how they changed through the project into the graph of emotions (see Appendix): The x axis of the graph is the time and the y axis is decided by each person, prospective teacher, for representing his/her own emotional change. Each person divided the graph by the periods for describing his/her emotional changes and the graph was explained with the periods by him/her. Thus, up and down of each graph is interpreted by each person's commentaries.

Even if each person's y axis meaning is very different, the phases are well reflected on their graphs (see Appendix: The periods ①~⑤ are rewritten in relation to Phase 1~5, not as same as original periods written by the persons.). In relation to the phases, graphs were categorized as follows: Like the graphs of Appendix 1, two persons' emotional changes are clearly related with the phases. Like the graph of Appendix 2, two persons' emotional changes did not exist phase 1 but other phases are matched with the graphs. They did not recognize phase 1 as a part of project because it was lectured by the second year students. Then, those four persons are clearly related with the phases. Like the graph of Appendix 3, three persons drew their growth of emotion and the highest emotional response is at the lesson implementation phase 4. Like the graph of Appendix 4, three persons connected Phase 2 and Phase 3 because they felt a very strong interest to read historical text as different mathematics and found their original subject matter for their Lesson Study from their readings. Like the graph of Appendix 5, two persons drew a valley at Phase 3 because they could not easily develop appropriate subject matter for teaching in classrooms. Other two persons' graphs are not clearly related with phases: One of them drew a gradual going up the graph and specially grew up at Phase 4 because he/she finally found strong mathematical interest in his lesson content. Another person drew just down after phase 1 because he/she chose the most difficult text, and felt strong difficulty in reading. He/She did not understand it well at the lesson implementation. He/She commented these kinds of mathematics are very far from school mathematics. All fourteen persons described their first impressions of projects in Phase 1 as interesting activity because they did not know school mathematics with historical text and how to use technology in mathematics. At the same time, even if teacher educator and second graders explained difficulty to read historical text and to develop subject matter from it, they could not imagine what they are and how hard they are to do.

2. An interpretation of a case

Even if we can analyze most of graphs in relation to phases, each prospective teacher's experience is very different. The explanations of periods described by each person are just their experience. Following figure is translated in English from one of Appendix 1. Handwritten numbers of ①~⑤ on the x axis are original descriptions of periods and they match to phases, clearly in this case. Here we interpret this person's emotional experience in the following way (Masami Isoda. 1998, 2000, Maitree Inprasitha, 2001): Depending on emotional theory by George Mandler (1984) based on the Piajetian cognitive model, emotional arousal is related with obstacles and challenges, and results such as overcoming obstacles give positive emotional feed backs. This cognitive cycle until reflection is also reasonable from the educational meaning of experience described by John Dewey. Based on Mandler's meaning of emotional change, we can interpret one down-up in the graph recognized as a strong experience.



Picture 1. A Case of one prospective teacher's experiences in the project

In this case, we analyze personal experience as follows: In period ①, this person (P) felt fun but did not have strong experience. P participated as a student in second year students' lessons and just enjoyed to learn last year's project. In period ②, there are two strong experiences (two down-ups). P began to read historical text and met the difficulty. P got some understanding of the text but did not understand it well. Then P found related two Japanese translation books and other supplementary books for trying to understand deeply. In period ③ and ④, there are intersections because P continued to develop materials during lesson implementation. P did not know how to develop materials from historical text but finally P developed: the strong experience of period ③. P felt anxiety to conduct the lesson but P implemented: the strong experience of period ④. In period ⑤, there is a deep valley after lesson implementation. It is a strong experience because P did not know how to write the report of the lesson. Next small down-up is developing the web site and P did not know the way also.

3. Didactical meaning of each phase for prospective teacher education

Even if there are two cases which did not well change the graphs in relation to phases, other twelve cases' graphs were explained in relation to phases. Their comments such as the ones seen in the case of figure 1 implicated each phase's didactical meaning for prospective teacher education. For clarifying didactical meaning of phased based on their comments, we would like to framework for interpretation of these data. Hans Nilse Jahnke (1994) used double circles for explaining historian's activity'Hermeneutics' in mathematics. First circle represents mathematician's activity on history and second circle represents historian's activity such as interpreting historical texts and asking why mathematicians did so. His model well represents the difference of mathematician's perspective and historian's perspective. Jahnke's double circles explain an activity of Phase 2. Here, we would like to expand his model to the field of teacher education for explaining nesting features of developing teachers' perspective 'kodomo womiru me' in the case of this Lesson Study project.

Figure 2-1

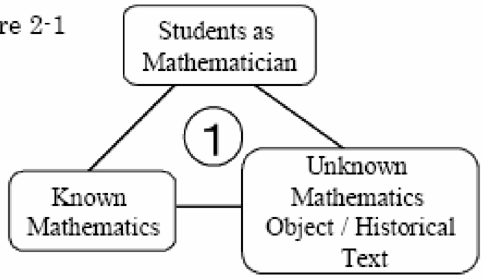


Figure 2-2

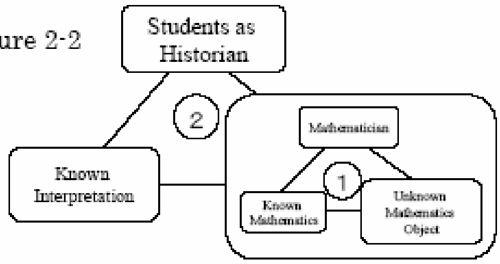


Figure 2-3

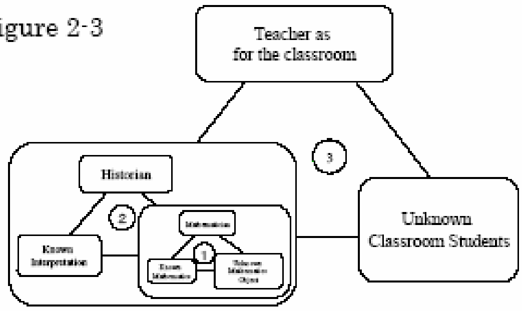


Figure 2-3*

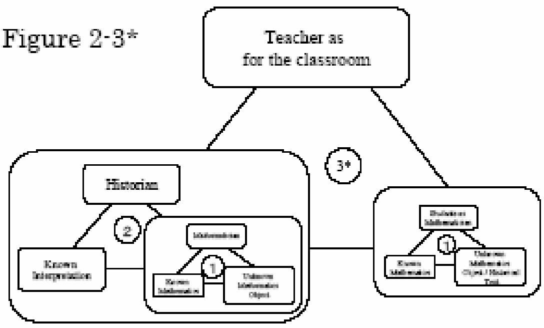


Figure 2-4

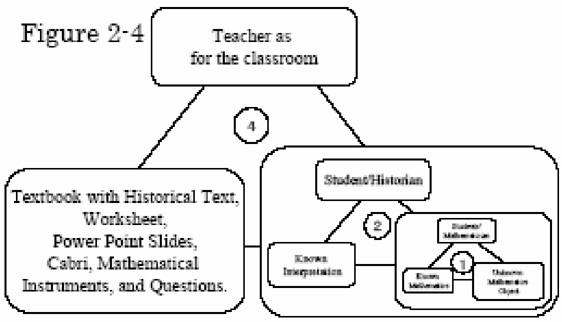
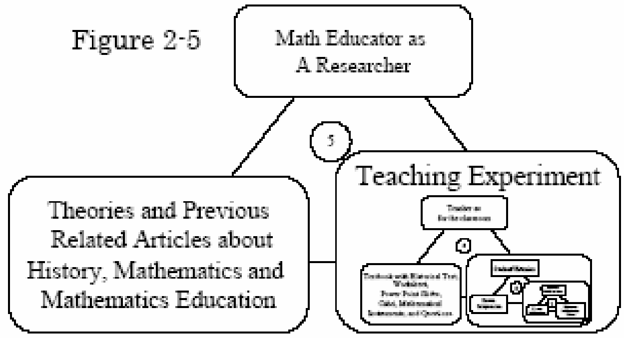


Figure 2-1 explains Phase I activity. Prospective teachers who are participating in the project enjoyed past project's lesson as students. They explored unknown mathematics originated from historical textbooks but reconstructed with educational questions by known mathematics.

Figure 2-2 explains phase 2 activity. They began to interprethistorical texts with known interpretations and were astonished with their differences when compared with today's mathematics. Figure 2-3 explains phase 3 activity.

They began to develop subject Matter. Before the project, they had experience of teaching with existed textbooks and it is the first experience for them to develop the textbook of totally new subject. From historian's activity on figure 2-2, they have to develop students activities with questions for the interpretations of textbook and they have to develop their aims of their lesson study project through thinking about what students can learn from their developed activities (figure 2-3*). It is very difficult for them because of their past experience of mathematics teaching is only related with mathematical problems but in this project, they have to make historical questions at the same time. Figure 2-4 explains Phase 4 activity. Finally, they had developed materials at phase 3 and then, they tried to conduct students' activities like mathematicians and historians. Figure 2-5 explains Phase 5 activity. They reflect on both of the teaching experiment of Phase 4 and all process of the project and redefine their research questions depending on what they did and analyze it with references.

Figure 2-5



Based on the analysis, we conclude the following didactical meanings on each Phase for prospective teacher education.

<u>Didactical Meaning of Phase 1</u>: It functioned to know the activity in the lessons through enjoying lessons in past projects like students. Even if teacher educator and second graders explained what the project is and what is necessary to do, such as questionings to classroom students, students, prospective teachers, could not imagine really the meaning because they still work as students who participate in the lessons.

<u>Didactical Meaning of Phase 2</u>: It functioned to know historian's activity such as constructing the meaning through the interpretation of historical texts. Many students felt difficulty to read historical texts at first, then they were astonished with the difference between today's mathematics and historical mathematics.

<u>Didactical Meaning of Phase 3</u>: It functioned to know developing subject matter as for students' activity with historical text and technology. Some students met strong difficulties for developing classroom materials. At the beginning, many students could imagine the textbook of mathematics history and could not develop educational questions through which students can explore historical texts.

<u>Didactical Meaning of Phase 4</u>: It functioned to know conducting the lessons. Many students were scared to conduct. For knowing how to, they practiced with each other before their lessons and expected students' activity based on their questions and reactions from students.

<u>Didactical Meaning of Phase 5</u>: It functioned to know how to write the research paper based on their teaching experiments.

4. Conclusion: A nesting feature of developing teachers' perspectives

These didactical meanings with figure 2-1 to 2-5 illustrate the process how prospective teachers possibly develop teachers' perspectives in this Lesson Study project. In this project sequence, phases are constructed like nesting structures. Every teacher's education subject matter functioned to use previous experiences from different perspectives. For enhancing different meanings of perspectives, we use the word 'role' as follows.

Role of Phase 1: Like mathematician
Role of Phase 2: Like historian
Role of Phase 3: Like textbook author
Role of Phase 4: Like master teacher
Role of Phase 5: Like math-educator

We conclude that the case treated various teachers' perspectives such as mathematician, historian, textbook author, master teacher and math-educator. The sequence of Lesson Study project has nesting structures to reflect previous activity from other view points in roles. This process illustrates one of possible way to develop teachers' perspectives.
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Appendix