Language Factors and Their Relevance in Problem Posing and Problem Solving in Primary Mathematics and Science Classrooms

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Abstract

It is often assumed that, apart from specialised terms, language has little to do with the teaching and learning of mathematics and science. A "key word" approach is often used to assist students to solve word problems. Often, teachers and textbook writers consciously simplify the wording of problems, or eliminate as many words as possible. This paper will show that such approaches are unwise, pedagogically. The paper will also demonstrate how sophisticated language skills are needed if children are to develop sound problem posing and problem solving skills. Results of research in the area will be summarised, and a framework for discussing language factors presented. A hands-on approach will be adopted in the presentation, with opportunities for participants to create and solve mathematics and science problems.

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Introduction

When children learn mathematics and science at school, they learn it through the medium of language. The teacher talks to the class, and the students talk to each other and to the teacher. They need to be able to understand what is said, and they need to be able to respond both verbally and in writing, as well as in their actions. Students are also expected to be able to read their mathematics and science textbooks, as well as to look up information about mathematics and science in the library or on the internet. Although no-one would question that language is important for these tasks, it is only comparatively recently that research has been carried out to help understand possible relationships between language factors and mathematics and science learning.

Although it is clear that all mathematical and scientific communications make use of ordinary language, the fact that the language we use every day is laden with mathematical and scientific terminology is often overlooked. Consider the following passage, taken from the first page of the website (National Parks, Malaysia, 2003).

It would be difficult to overstate the attraction of Malaysia for anyone who appreciates the natural world. Its primal forests, ranging from shoreline mangrove to mountaintop oak, are of the sort that most of the world now knows only in myth. Although Malaysia's size is similar to that of Norway, natural trees and forests cover almost three quarters of the land, an area equivalent to almost the entire United Kingdom. One can walk for hundreds of miles in Malaysia under a continuous canopy of green, marveling at an abundance of plant and animal species equaled by no other location in the entire world. A single half-kilometre plot of land in Borneo's lowland dipterocarp forest, for example, may well contain more than eight hundred different species of trees alone, a stunning degree of variety that pales, however, in comparison to the profusion and diversity of flowers, birds, ferns, and insects.

The passage on the website is intended for a general audience, so it contains very little specialised language. At the same time, the writers wanted to fit as much interesting information into the leading section of the website as possible. It is therefore worth analysing what level of scientific and mathematical understanding would be needed to be able to read, understand, and enjoy the passage above.

1. The word "primal" implies the notions of "old", "original", and "belonging to a bygone era" - in other words, originating a very long time ago.
2. The reader is being asked to visualise the terrain, "from shoreline mangrove to mountaintop oak." Visualisation is a skill which cannot be taken for granted in the solving of mathematical and scientific problems, or in the reading of such passages.

3. Two comparisons of the size of Malaysia with other countries (Norway and the United Kingdom) are made. The reader therefore needs to have some knowledge of the geography of the world, and an ability to be able to relate sizes on the scaled map to the size of large land masses.
4. Fraction knowledge is important. Many people, young and old, have a poor understanding of fractional relationships. They may understand, for example, how to select half of the mangos in the basket, or to share an apple equally between four people. They may also have used area models to learn fraction concepts by sharing a pie equally among six people. But this statement asks the reader to develop a mental picture of what it means to cover three-quarters of the land of Malaysia with natural trees and forests. Fractions never looked like that in school.
5. In Malaysia, distances are normally quoted in kilometres. Yet the expression "one can walk for hundreds of miles" has been used. To those of us who have lived through decimal conversion, the "old" terminology used here sounds much more comfortable than the more technically correct statement "one can walk for hundreds of kilometres." The terminology "hundreds of miles" is a comfortable and familiar term in its own right, implying far into the distance. But would *all* readers know that this is what the authors probably intended? Incidentally, would readers realise that this is one case when "hundreds of miles" probably means the same things as "hundreds of kilometres"?
6. Reference is made to a "single, half-kilometre plot of land." This assumes an understanding of a colloquial way of referring to a square plot of land measuring half-a-kilometre by half-a-kilometre. It also assumes that the reader will know that someone didn't go into the Borneo jungles and mark out one small plot of land. The passage implies the use of a statistical sampling procedure which might be used to estimate the number of different populations inhabiting the forest.
7. The word "dipterocarp" is likely to be unfamiliar terminology to many readers, but would have been included for scientific accuracy, and to help people know the term in its correct usage. Another reason for its inclusion may have been in the hope of adding a level of mystery and the unknown to a familiar environment.
8. Finally, the reader is meant to acknowledge that eight hundred different species represents a large number of species of trees – thus requiring not only knowledge of the number system, but also some understanding of the environment around them.

This passage serves to illustrate how predominantly simple language can incorporate deceptively complex mathematical and scientific concepts and conventions.

The Role of the Teacher

One of the crucial roles of a teacher is to assist learners to acquire the formal languages of mathematics and science, in both receptive and expressive modes. In the receptive mode, students need to be able to listen, to read, to observe, and to interpret, but in the expressive mode, they also need to be able to express themselves verbally to the teacher and to their peers, to write out their solutions to problems and explanations of scientific observations, to act out solutions to problems and to carry out field and laboratory investigations. In short, they need to have the opportunity, support and skills to express themselves in a variety of ways. Del Campo and Clements (1990) set out the ideas shown in Table 1 to help teachers understand the importance of including all modes of communication in their classrooms.

Table 1.
Receptive and Expressive Modes of Communication (from Del Campo & Clements 1990, p. 59)

Language Mode	Receptive Language (processing someone else's communication)	Expressive Language (your own language)
Spoken	listening	speaking
Written	reading	writing
Pictorial	interpreting diagrams, pictures	drawing
Active	interpreting others' actions	performing, demonstrating
Imagined	-	imagining*

* Note that 'imagining' has been included as an expressive form of language because by imagining, one can communicate with oneself.

Students model their language and actions on the language and actions of those around them. Learning the languages of mathematics and science, however, does not just involve learning the vocabulary. Words and phrases used in every-day language often take on a different meaning when used in the contexts of science or mathematics classrooms. In addition, the genre of mathematics and science textbooks both differ from the genre of everyday conversation.

In the section which follows, four specific language factors which affect the teaching and learning of mathematics will be considered.

Four Specific Factors

Deep Understanding of Mathematical and Scientific Vocabulary is Needed.

Although it may be stating the obvious to emphasise that students need to understand specialised terminology in mathematics and science, research suggests that such terminology is not necessarily understood, even by students who can use the particular word or words (see, for example, Earp & Tanner, 1980; Gardner, 1974; Malone & Miller, 1993; Nicholson, 1979). Words that students find difficult include words such as "contribute" and "external", "limit" and "process" which have specialised meanings in mathematical and scientific contexts. Students also find words like "diameter" and "perpendicular" difficult, even though these are words more normally associated with their mathematical meanings.

Although the results of research about vocabulary are of importance to curriculum developers, teachers, textbook writers, and those responsible for constructing assessment items, what is essential is that students develop a deep understanding of the meaning of mathematical and scientific vocabulary. It is not sufficient for a student to be able to *interpret* words like "contrast" "vertical", and "function" correctly; students need to be able to *use* and *apply* such words appropriately in a range of contexts, and via different modes of communication, before it can be said that they *understand* their meaning. It is relatively simple for students to develop an *instrumental* understanding of vocabulary; the challenge for the teacher is to help students develop *relational* understanding (Skemp, 1976).

Semantic Structure is More Important Than Vocabulary.
In a classic study by Riley, Greeno and Heller (1983), it was found that problems which seem to involve the same arithmetic numbers and operations can differ substantially in their difficulty for children. For example, the following two word problems both give rise to the equation $3 + 5 = 8$, but the second is much harder than the first for young children.

1. Siti had 3 durian. Abdul gave her 5 more durian. How many durian did she have then?
2. Azri gets home from work at 3 o'clock. This is 5 hours before Jaya gets home from work. What time does Jaya get home from work?

Research involving large samples in Australia and Papua New Guinea by Lean, Clements and Del Campo (1990), and in Africa by Adetula (1990), suggests that semantic and syntactic structures have an overriding and universal influence on problem difficulty. Similarly, MacGregor (1991) asked 235 Year 9 students in Australia to answer the following three questions, and found that the percentages correct were 34.5%, 28.1%, and 33.2%, respectively, with over half of the total responses to the first of these questions containing expressions like "$y \times 8$," "$8y$," and "y^8."

1. "The number y is eight times the number z." Write this information in mathematical symbols.
2. "s and t are numbers. s is 8 more than t." Write an equation showing the relation between s and t.
3. "The Niger River in Africa is y metres long. The Rhine in Europe is z metres long. The Niger is three times as long as the Rhine." Write an equation that shows how y is related to z.
   (MacGregor, 1991, p. 92)

MacGregor (1991, p. 98) commented that the variety and frequency of errors were unexpected since the syntax was simple and straightforward. However, I believe that the difficulty was clearly associated with semantic structure, and that mathematics educators need to pay far more attention to assisting children to *comprehend* mathematical word problems. Comments in mathematics teachers' journals on such issues (see, for example, Stiff, 1986) have too often been at a superficial level in that they have not answered the question why many students do not seem to be able to keep track of information presented in word problems. Teachers need to realise that usually the reason why children do not keep track of the information is that they are unable to cope with the semantic structures of the questions.

Although this problem associated with semantic structure permeates all of school mathematics and science, it remains largely unrecognised by teachers and curriculum developers. However, having said this, some professional development programs (for example, *Cognitively Guided Instruction* initiated by Fennema, Carpenter & Peterson, 1989) emphasise the importance of semantic structure on problem difficulty. Evaluations of these programs indicate that, once teachers become aware of the central importance of semantic structure, they are able to modify their teaching practices so that their students are able to improve dramatically in their comprehension of mathematics word problems.

In mathematics and science classrooms, it is common for teachers to use the so-called "key word" approach whereby they encourage students to associate particular operations with certain words. Thus, for example, comparative terms such as "more than", heavier than", "longer than" are to be associated with addition, and terms such as "less than" and "before" are to be associated with subtraction. However, such a teaching strategy is seriously flawed. For example, consider the problem: "My container holds 200 millilitres less than your container. If my container holds 1 litre, how much would yours hold?" A key word approach would give an incorrect answer. From a cultural perspective, Harris's (1987) work made clear that the formal language of comparison is very much a cultural matter, with many Australian Aboriginal groups thinking about measurement of quantities in quite different ways from, say, Western cultural groups. The semantic structures of the formal language of comparative terms can vary greatly between languages and cultures.

Strong Links Need to be Established Between Students' Personal Worlds on the One Hand, and Formal Mathematical and Scientific Language and Skills on the Other.

Research by Clements and Lean (1988) and MacGregor (1991) has established that many students simply do not understand what they are asked to do in mathematics classes because the questions are unrelated to their personal worlds. For example, Clements and Lean (1988) found that Grade 5 children who could correctly find the value of 7/11 x 792 were not able to give an interviewer one-quarter of 12 stones. Yet, they had no trouble sharing 12 objects equally among four friends.

Once again, the research literature tends to suggest that if teachers are conscious of the need to establish links, and devise learning environments in which linking is of paramount importance, then their students' comprehension of word problems is considerably enhanced, and their receptive and expressive performance improved (Clements & Del Campo, 1989). Teachers' roles in establishing learning environments which facilitate such links become especially challenging when a range of different cultures are present in the same classroom (see for example, Harris, 1987).

Problem Comprehension and Problem Transformation are Major Sources of Errors on Mathematics Word Problems.

The major research supporting this statement has been based on what has become known as the Newman error analysis procedure (see Clements, 1980), which is summarised in Figure 1. Much has been written on this procedure which has been widely used in mathematics and science education research in Southeast Asia (see, for example, Ellerton & Clements, 1996; Jiminez, 1992; Mohidin, 1991; Singhatat, 1991) as well as in Australia and Papua New Guinea (Ellerton & Clarkson, 1992).

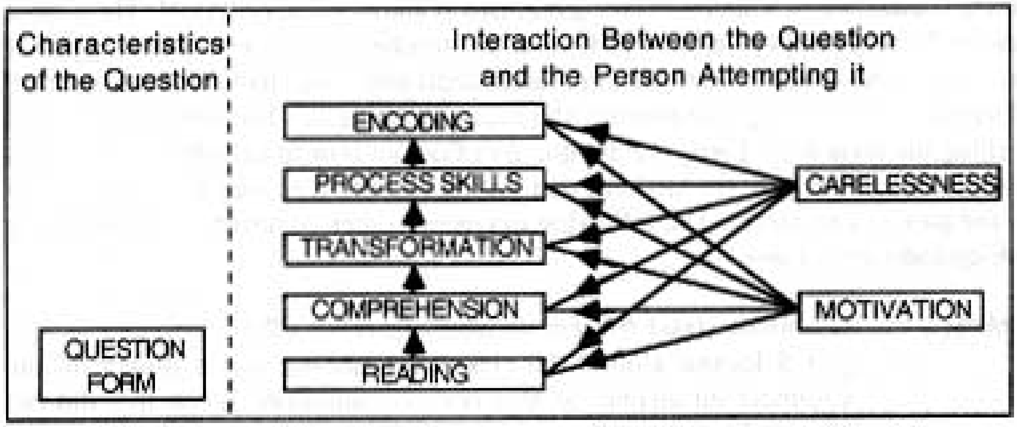

Figure 1. The Newman hierarchy of error causes (from Clements, 1980, p. 4).

With the Newman procedure, students who have already attempted to solve a mathematics word problem are asked a sequence of questions which are aimed at seeing whether they can:

1. Read the problem;
2. Comprehend what is read;
3. Carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy;
4. Apply the process skills demanded by the selected strategy; and
5. Encode the answer in an acceptable written form.

Newman also allowed for "careless errors" and errors due to a lack of motivation on the part of students (see Newman, 1977, 1983).

Newman error analysis research by Marinas and Clements (1990), Singhatat (1991), and Clements and Ellerton (1992), in Southeast Asia reported that about 70% of errors made by students on standard word problems could be attributed to a lack of comprehension or to an inability to select an appropriate sequence of operations (that is, in Newman's terms, to carry out the required "Transformation"). The strength of such data suggests that one of the urgent agendas of mathematics education research is to establish ways of addressing this state of affairs. Above all else, Newman research points to the students' lack of a deep understanding of mathematical vocabulary, semantic structure, and the absence of links between the students' formal language and mathematical skills, and their personal worlds. In other words, the Newman research procedure provides a framework for investigating each of the major themes discussed above (Clements & Ellerton, 1996). Clements (1999) argued that mathematical modelling should be present whenever a learner attempts to solve a real-life problem or a mathematics word problem. In that sense, Newman's transformation often involves mathematical modelling. Often, this process of transformation or mathematical modelling is not evident in mathematics classroom discourses.

An Episode in a Grade 7 Mathematics Classroom

The importance of language factors is illustrated by the following episode (discussed in Ellerton, 1999a) from a middle-stream Grade 7 classroom in a regional primary school. The Grade 7 class was a middle-stream group in a regional primary school. There were 20 students present (10 boys and 10 girls), and the teacher (Mr S), who had a degree in mathematics, was young, enthusiastic, and dedicated. He had obviously spent considerable time preparing a lesson on "the area of a triangle." The lesson began with Mr S telling the class that "Today we are going to find out how to calculate the area of a triangle." He then reminded the students that, in their last lesson, they had learned how to find the area of a rectangle. The following interactions then occurred (**E1** refers to the first sub-episode, and so on):

E1: Mr S: [To whole class] *What is the area of a rectangle?*
[Mr S looked around the classroom, hoping that a student would volunteer an answer to this revision question. When this did not happen, Mr S decided to ask a particular student.]
Mr S: *Julie, what do you think? What's the area of a rectangle?*
Julie: [surprised at being asked] *Um, um, . . . 180?*

In this sub-episode, the teacher had apparently decided that it was his responsibility to link the new topic with what was done in the last lesson. His initial, rather vague question was not understood by the students. It was not clear whether he meant "What is the formula for the area of a rectangle?" or whether he was asking what was the students' concept of the area of a rectangle. In any case, he assumed (almost certainly, falsely) that the students all understood what he meant by his initial question. Julie, though, had heard him use the word "triangle" in his introduction to the lesson, and on being asked to provide an answer to the teacher's question, stammered, "180?" Presumably she knew that 180 had something to do with triangles, and she had focused on the properties of a triangle rather than on those of a rectangle.

Mr S then decided to go on with the main theme for the day:

E2: Mr S: [To whole class] *You'll remember that in the last lesson we found that the area of a rectangle is given by $A = L \times B$.* [Mr S then sketched a rectangle on the blackboard, and labelled it as shown in Figure 2] Above the rectangle he wrote:

A = L x B

Area of rectangle = Length x Breadth

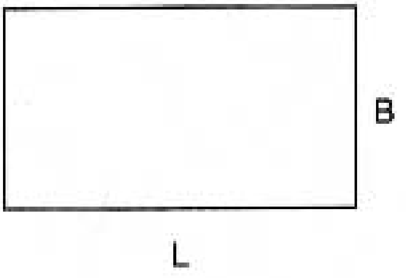

Figure 2. Area = Length x Breadth

He then continued:

E3: Mr S: [To whole class] *Any triangle can be regarded as half a rectangle. Have a look at this.* [He drew a triangle within the rectangle he had previously drawn (the triangle having the same base as the rectangle), and then dotted in the altitude of the triangle - see Figure 3] He continued:

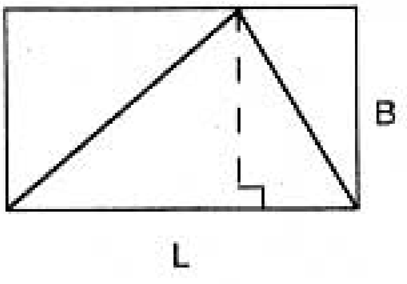

Figure 3. Establishing the area of a triangle.

Mr S: *You can see that the area of the triangle is half that of the rectangle because that bit there*(pointing to the left part of Figure 3) *is half of that rectangle, and that bit there* (pointing to the right part of Figure 3) *is half of that rectangle. So the area of the whole triangle is half the area of the whole rectangle. Therefore the area of the triangle is that length there* (pointing to the base of the triangle) *multiplied by that length there*(pointing to the altitude), *all divided by two. So, if we have a triangle like this* . . . [draws a triangle with a horizontal base, dots in an altitude, and labels the base and altitude with the pronumerals b and a, respectively - see Figure 4], *then* . . . [writes, next to the triangle]:

$A = (b \times a) \div 2$

Area of triangle = (Base x Altitude) ÷ 2

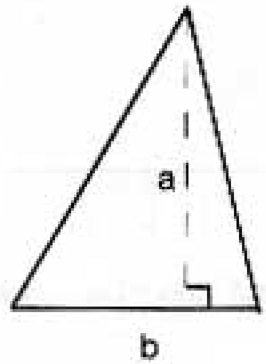

Figure 4. Area = (base x altitude) ÷ 2

He then continued:

E4: Mr S: *Let's try an example and you'll see what I mean . . .* [draws Figure 5] *Let's find the area of this triangle. How would you do it, Bill?*

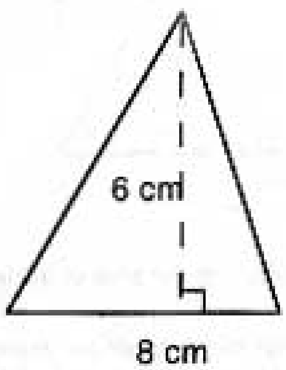

Figure 5. "Sir, it's wrong!"

Bill: [surprised at being asked]. *Er . . . What was that sir?*
Mr S: *How would you find the area of this triangle?*
Bill: *Sir, it's wrong!*
Mr S: *What do you mean, it's wrong, Bill?*
Bill: *How can that be 8 and that 6, when the side that's 8 is less than the side that's 6?*
Mr S: *OK, OK! The drawing wasn't to scale.* [He redraws the diagram as in Figure 6] *OK, Bill. What's the area of the triangle, now?*

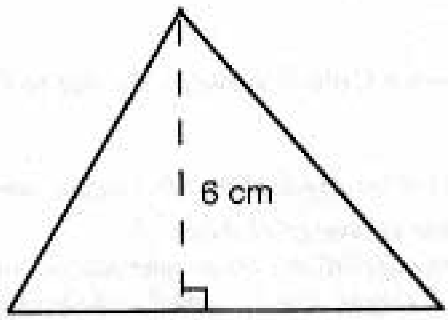

Figure 6. "Find the area of this triangle."

Bill: *Mmm. Forty-eight?*
Mr S: *How did you get 48?*
Bill: *Eight sixes are forty-eight.*
Mr S: *Almost. But with triangles you've got to halve that. So what would be the area, Bill?*
Bill: *What do you halve? Do you halve the eight and the six, or do you just halve one of them?*
Mr S: *You just multiply the two and then you halve the lot.*
Bill: *Alright. So the answer is half of 48. That's 24.*
Mr S: *Almost right. You forgot to put in the unit. It's cm ^{2}. So the right answer is 24cm ^{2}.* [Looking at the class] *Any questions?*

Mr S then asked students to do 20 exercises from a page in their mathematics textbook. He told them they could use a calculator, and that all they needed to do was multiply the base by the altitude and divide by 2. "Don't forget to put in the unit squared," he added. Each of the 20 exercises was based on an acute-angled triangle, with a base drawn horizontally. In each case, the length and the corresponding altitude were shown.

A researcher who observed and made notes of the lesson decided to ask a student (Cathy) about what she had done to obtain the answer 17.5 cm^2 for the area of a triangle with base 7 cm and altitude 5 cm.

E5:
Researcher: *How did you get the 17.5?*
Cathy: *I timesed 7 by 5 and divided by 2. Like this . . .* [Demonstrated how to get 17.5 on a calculator.]
Researcher *Well, what's the 17.5 got to do with the triangle?*
Cathy: *It's the answer. It's what you get when you times 7 by 5 and then you divide by 2.*
Researcher: *But what does the 17.5 mean for the triangle? What is there about the triangle that's 17.5?*
Cathy: *17.5 is the answer; it's what comes up on the calculator.*

Mr S was nearby and overheard Cathy's answers. He said to Cathy, "17.5 cm^2 is the *area* of the triangle."

Researcher:	[To Cathy] *What do you think Mr S means when he says that the triangle has anarea of 17.5 cm^2.?*
Cathy:	[Somewhat puzzled] *It's the answer you get when you do the timesing. It's right, 'cos I checked at the back of the book.*

In the above episode, a young enthusiastic mathematics teacher with a relational understanding (Skemp, 1976) of how the areas of triangles and rectangles are inter-related (see sub-episodes **E2** and **E3**), has as his main agenda the aim of helping his students to understand the relationship just like he does. The difficulty is that some of his students (a) have no idea of what the term "area" means (**E1, E5**); (b) do not know what the formula for the area of a triangle $A = (b \times a) \div 2$ really means (other than it instructs them to do a multiplication followed by a division when they are asked to find an area) (**E5**) ; and (c) most importantly, were unable to follow the sequence of words used by the teacher when he tried to explain why the area of a triangle is half the area of a rectangle (**E4, E5**).

In more general terms, the teacher does not appear to be aware of the extent of his students' comprehension difficulties. For their part, the main agenda of many of the students was to try to look for words, symbols, diagrams and sequences of actions (on a calculator, for example) that would help them get the right answer. Such students are not really worried if they fail to understand what the teacher is getting at - they believe that if they can get correct answers, then they understand. If the students are subsequently asked to do mathematics tests in which the emphasis is on instrumental knowledge and skills rather than on relational understanding (Skemp, 1976), then their false belief that they really do understand is reinforced.

Message of Research into the Discourses of Mathematics/Science Classrooms

Some readers may feel that the above episode represents the efforts of a young teacher who almost certainly will become more aware of his students' levels of understanding as he becomes more experienced. However, research into the discourses of mathematics classrooms, carried out in various countries around the world, suggests that many of the main elements of the above episode are very common in mathematics classrooms, at all levels.

Erlwanger's (1975) classic study of mastery learning classrooms in the United States, showed that many students who had passed tests and examinations had very little understanding of the mathematics that had been tested. The German researchers, Bauersfeld (1980) and Voigt (1985), have showed that there are hidden dimensions of mathematics classrooms which result in students simply not understanding the language used by teachers, textbooks and examiners - yet many of these students are able to pass examinations merely by applying rote-learned rules in response to standard cues - they learn to "play the game." It is these hidden dimensions which define the *culture* of mathematics classrooms, a culture which comprises unstated and even unconscious agreements between teacher and students. These agreements permit a form of mathematics education to proceed, even though many students find themselves listening for key words, phrases, and rules which will enable them to pass tests and examinations.

Voigt (1985) carried out a micro-analysis of four lessons in which a teacher routinely introduced new mathematics tasks by asking open-ended questions. Voigt's analysis revealed that, although he thought his questions were encouraging constructivist learning, unknowingly, by means of implicit markers and non-verbal cues, he had funneled the students towards the solutions he had had in mind all along.

The radical constructivist view point is that it is impossible to transmit knowledge from one person to another by written or spoken language – learners will interpret what is read and heard, and will construct their own personal meaning (von Glasersfeld, 1983). Or, as Wheatley (1991, p. 10) puts it, "knowledge is not passively received, but is actively built up by the cognizing subject," and ideas and thoughts "cannot be communicated in the sense that meaning is packaged into words and 'sent' to another who unpacks the meaning from the sentences." Thus, Wheatley states, "our attempts at communication do not result in conveying meaning but rather our expressions *evoke* [Wheatley's emphasis] meaning in another, different meanings for each person."

In a section entitled "Transmission Under the Guise of Reflection and Invention," Clements and Ellerton (1991) analysed a transcript which showed a teacher who wanted her students to sense that a piece of mathematics was aesthetically pleasing and beautiful, failed to do so largely because the patterns in the mind of the teacher were simply too subtle for the Grade 7 pupils, even though the teacher did her best to establish a learning environment in which the students might have been expected to construct the patterns. There was a mismatch between the teacher's understandings and language, the mathematics itself, and the students' understandings and language.

This raises the issue of whether there are any general characteristics of mathematics classroom learning environments which are likely to reduce or even eliminate the mismatch between curriculum, teacher and learners. This issue has been addressed at length by radical constructivist mathematics educators such as Cobb, Yackel and Wood (1992), Steffe (1990), and Pateman and Johnson (1990). Here, it suffices to present the list of five qualities of mathematics classroom environments for which Cobb (1990, pp. 209-210) has claimed there is research support:

1. Learning should be an interactive as well as a constructive activity - that is to say, there should always be ample opportunity for creative discussion, in which each learner has a genuine voice;
2. Presentation and discussion of conflicting points of view should be encouraged;
3. Reconstructions and verbalisation of mathematical ideas and solutions should be commonplace;
4. Students and teachers should learn to distance themselves from ongoing activities in order to understand alternative interpretations or solutions;
5. The need to work towards consensus in which various mathematical ideas are co-ordinated is recognised.

Although many teachers of mathematics would claim that they already incorporate all five of these points in their classrooms, all too often the rhetoric of mathematics teachers and the realities of what transpires in their mathematics classrooms do not bear much resemblance to each other (Desforges, 1989). This list from Cobb (1990) at least provides a basis for curriculum development and associated professional development programs. There is certainly a tension between the need for teachers to maintain authority and direction in their classrooms, to cover required content, and yet to be sufficiently flexible to encourage students to construct their own mathematical meanings by being engaged in genuine problem-posing and problem-solving situations (Ellerton & Ellerton, 1987).

Development of a Model Relating Language and Mathematics

During the 1970s and 1980s mathematics educators became increasingly aware of the diverse strands of research that are concerned with the interface between language, mathematics, and mathematics learning. The model put forward by Ellerton (see Ellerton, 1989; Ellerton & Clements, 1991; Ellerton & Clarkson, 1996) was developed in response to the obvious need to develop links between the various research thrusts already well established in the mathematics education literatures. The model is summarised in Figure 7, and reflects the complexity of the multi-faceted relationships between mathematics, language, the mathematics classroom, and the underlying cultural context.

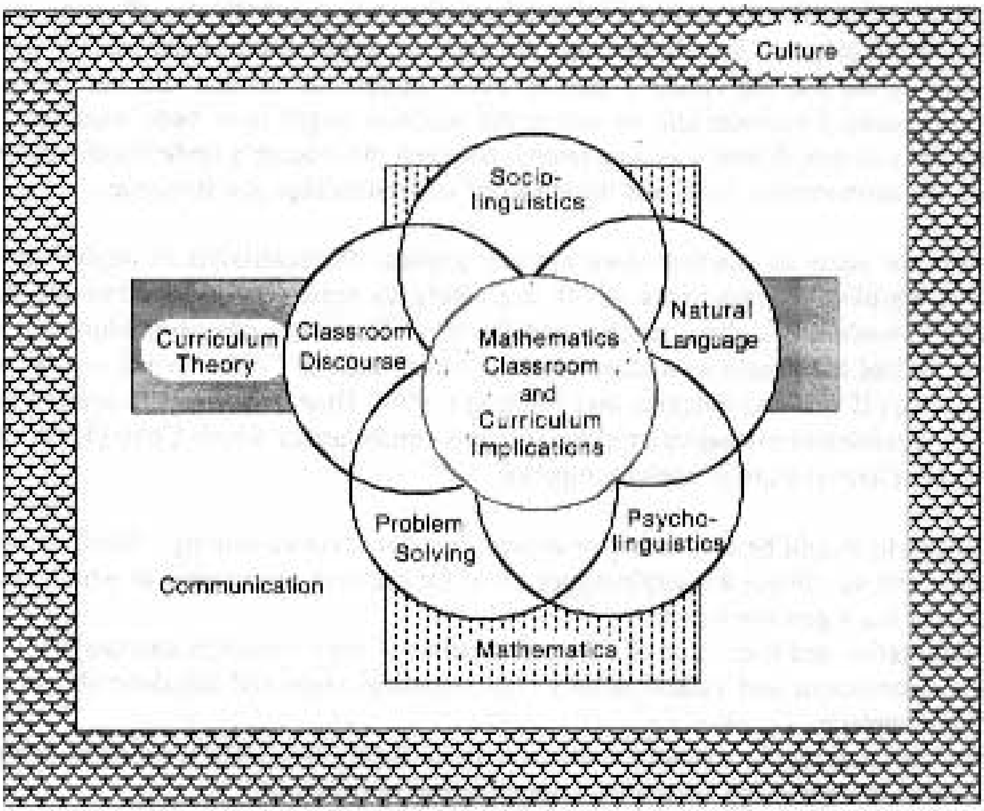

Figure 7. The interface between mathematics and language

Although the definitions of, and relationships between, the components of a model such as this are open to debate, the theorising of the links between the various areas of research in language and mathematics is seen not only as an important way of unifying and directing research, but also as something likely to influence curriculum development as well as the teaching and learning of mathematics. A similar model could be constructed for facilitating discussion about the teaching and learning of science (Ellerton, 1999b).

Concluding Comments

In the past, language education and mathematics/science education researchers have tended to work separately, being largely unaware of the efforts of those in closely related fields of endeavour. For greater detail which synthesises the interface between mathematics and language suggested by the theoretical model presented in Figure 7, the reader is referred to Ellerton & Clements (1991). In this present paper, my aim has been to provide a survey of some of the factors and approaches that research seems to suggest are of central importance if the teaching and learning of mathematics are to be improved.

The following points are presented to serve as a springboard for further research and for the development of more appropriate mathematics curricula and professional development programs for teachers of mathematics.

1. *Communication mismatches between teachers and students, and hidden dimensions of mathematics and science classroom discourses, are of fundamental importance in explaining why many students find it difficult to learn these subject areas.* Children's understanding is fundamentally influenced by the many hidden dimensions of mathematics and science classrooms; in particular, the complexity of common patterns often inhibit learning. Among the barriers to learning is the idea that it is somehow acceptable for teachers to use a higher level of language than students can appreciate. While teachers often have a relational understanding of what they are teaching, many students find themselves only able to acquire a surface level of understanding of what is intended by the teacher. In short, there is a communication mismatch.

2. *Teachers not only need to become more aware of the difficulty students experience in comprehending specific mathematical and scientific vocabulary, and the semantic structures embedded in the languages of mathematics and science, but they should also take steps to help their students to acquire genuine understanding of these languages.* Research is unambiguous on the point that many students simply do not understand what teachers, textbook writers and examiners are talking about. The communication mismatch referred to in point 1 derives from this lack of understanding. There is an old saying that every teacher should be a teacher of language, and research seems to suggest that the idea behind this saying is particularly important in mathematics and science classrooms.

3. *Establishing cognitive links is a fundamentally important curriculum issue in mathematics education.* Mathematics and science curricula need to be developed which are structured around the idea of linking learners' personal worlds with formal mathematical and scientific skills and with formal mathematical and scientific language.

4. *The need to assist learners to develop appropriate cognitive links is a vitally important professional development issue in mathematics education.* Once mathematics curricula which are structured around the idea of linking learners' personal worlds with their formal mathematical skills and with their formal mathematical language have been developed, then appropriately funded professional development programs aimed at helping teachers to establish learning environments that will promote linking, should be implemented.

References

Adetula, L. O. (1990). Language factor. Does it affect children's performance on word problems? *Educational Studies in Mathematics, 21,* 351-365.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics 11(1),* 23-41.

Clements, M. A. (1980). Analysing children's errors on written mathematical tasks. *Educational Studies in Mathematics, 11*(1), 1-21.

Clements, M. A. (1999). Language aspects of mathematical modelling in primary schools. In M. A. Clements & Leong Yong Pak (Eds.), *Cultural and language aspects of science, mathematics, and technical education* (pp. 363–372). Gadong: Universiti Brunei Darussalam.

Clements, M. A., & Del Campo, G. (1989). Linking verbal knowledge, visual images, and episodes for mathematical learning. *Focus on Learning Problems in Mathematics, 11*(1), 25-33.

Clements, M. A., & Ellerton, N. F. (1991). *Polya, Krutetskii, and the restaurant problem.* Geelong: Deakin University.

Clements, M. A., & Ellerton, N. F. (1992). Overemphasising process skills in school mathematics: Newman analysis data from five countries. In W. Geeslin & K. Graham (Eds.), *Proceedings of the Sixteenth International Conference on the Psychology of Mathematics Education* (Vol. 1, pp. 145-152). Durham, New Hampshire: International Group for the Psychology of Mathematics Education.

Clements, M. A., & Ellerton, N. F. (1996). *Mathematics education research: Past, present and future.* Bangkok: UNESCO. (250pp.)

Clements, M. A., & Lean, G. A. (1988). "Discrete" fraction concepts and cognitive structure. In A. Borbas (Ed.), *Proceedings of the Twelfth International Conference on the Psychology of Mathematics Education* (Vol 1, pp. 215-222). Veszprém, Hungary: International Group for the Psychology of Mathematics Education.

Cobb, P. (1990). Multiple perspectives. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 200-215). Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*(1), 2-33.

Del Campo, G., & Clkements, M. A. (1990). Expanding the modes of communication in mathematics classrooms. *Journal furMathematik Didaktik, 11*(1), 45-79.

Desforges, C. (1989). Classroom processes and mathematical discussions: A cautionary note. In P. Ernest (Ed.) *Mathematics teaching: The state of the art* (pp. 143-150). Barcombe (East Sussex): Falmer Press.

Earp, N. W., & Tanner, F. W. (1980). Mathematics and language. *Arithmetic Teacher, 29*(4), 32-34.

Ellerton, N. F. (1989). The interface between mathematics and language. *Australian Journal of Reading, 12*(2), 92-102.

Ellerton, N. F. (1999a). Landmarks: A metaphor for a theory of mathematics and science learning. Leong Yong Pak & M. A. Clements (Eds.), *Cultural and language aspects of science, mathematics and technical education* (pp. 354-362). Gadong, Negara Brunei Darussalam: Department of Science and Mathematics Education, Universiti Brunei Darussalam.

Ellerton, N. F. (1999b). Language factors affecting the learning of mathematics and science: A holistic perspective. Leong Yong Pak & M. A. Clements (Eds.), *Cultural and language aspects of science, mathematics and technical education*(pp. 59-68). Gadong, Negara Brunei Darussalam: Universiti Brunei Darussalam.

Ellerton, N. F., & Clarkson, P. C. (1992). Language factors in mathematics education. In B. Atweh & J. Watson (Eds.), *Research in mathematics education in Australasia 1988-1991* (pp. 153-178). Brisbane: Mathematics Education Research Group of Australasia.

Ellerton, N. F., & Clarkson, P. C. (1996): Language and cultural factors in mathematics teaching and learning. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 987-1033). Dordrecht: Kluwer.

Ellerton, N. F., & Clements, M. A. (1991). Mathematics in language: A review of language factors in mathematics learning. Geelong: Deakin University.

Ellerton, N. F., & Clements, M. A. (1996). Newman error analysis: A comparative study involving Year 7 students in Malaysia and Australia. In P. Clarkson (Ed.), *Technology in Mathematics Education* (pp. 186-193). Melbourne: Mathematics Education Research Group of Australasia.

Ellerton, N. F., & Ellerton, H. D. (1987). Mathematics and Chemistry problems created by students. In J. D. Novak (Ed.), *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics* (Vol. 3, pp. 130-136). Ithaca, NY: Cornell University.

Erlwanger, S. H. (1975). Case studies of children's conceptions of mathematics - Part 1. *Journal of Children's Mathematical Behavior, 1*(3), 157-283.

Fennema, E., Carpenter, T. P., & Peterson, P. (1989). Teachers' decision making and cognitively guided instruction. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 174-187). Geelong: Deakin University.

Gardner, P. (1974). Language difficulties of science students. *Australian Science Teachers Journal, 20*(1), 63-76.

Harris, P. (1987). *Measurement in tribal Aboriginal communities* (Second Edition). Darwin: Northern Territory Department of Education.

Jimenez, E. C. (1992). *A cross-lingual study of Grade 3 and Grade 5 Filipino children's processing of mathematical word problems.* Penang: SEAMEO-RECSAM

Lean, G. A., Clements, M. A., & Del Campo, G. (1990). Linguistic and pedagogical factors affecting children's understanding of arithmetic word problems. *Educational Studies in Mathematics, 12*(3), 267-299.

MacGregor, M. E. (1991). *Making sense of algebra: Cognitive processes influencing comprehension.* Geelong: Deakin University.

Malone, J., & Miller, D. (1993). Communicating mathematical terms in writing: Some influential variables. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), *Communicating mathematics: Perspectives from classroom practice and current research* (pp. 177-190). Melbourne: Australian Council for Educational Research.

Marinas, B., & Clements, M. A. (1990). Understanding the problem: A prerequisite to problem solving in mathematics. *Journal for Research in Science and Mathematics Education in Southeast Asia, 13*(1), 14-20.

Mohidin, R. (1991). An investigation into the difficulties faced by the students of Form 4 SMIA secondary school in transforming short mathematics problems into algebraic form. Penang: SEAMEO-RECSAM. National Parks, Malaysia (2003). http://www.geographia.com/malaysia/nationalparks.htm
Site accessed, 25 March, 2003.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39*, 31-43.

Newman, M. A. (1983). *Strategies for diagnosis and remediation.* Sydney: Harcourt, Brace Jovanovich.

Nicholson, A. R. (1979). Mathematics and language revisited. *Mathematics in School, 18*, 44-45.

Pateman, N. A., & Johnson, D. C. (1990). Curriculum and constructivism in early childhood mathematics: Sources of tension and possible resolutions. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 346-356). Hillsdale, NJ: Lawrence Erlbaum.

Riley, M. S., Greeno, J., & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.

Singhatat, N. (1991). *Analysis of mathematics errors of lower secondary pupils in solving word problems.* Penang: SEAMEO-RECSAM.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Steffe, L. P. (1990). Mathematics curriculum design: A constructivist's perspective. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 389-398). Hillsdale, NJ: Lawrence Erlbaum.

Stiff, L. V. (1986). Understanding word problems. *Mathematics Teacher, 79*(3), 163-165.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Récherches en Didactique des Mathématiques, 6*, 69-118.

von Glasersfeld, E. (1983). Learning as a constructivist activity. In J. C. Bergeron & N. Herscovics (Eds.) *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 41-69). Montréal: Psychology of Mathematics Education, North American Chapter.

Wheatley, G. (1991). Constructivist perspectives on science and mathematics learning. *Science Education, 75*(1), 9-21.

PRESENTATION REPORTS: MATHEMATICS PAPERS

Title : **Language Factors and their Relevance in Problem Posing and Problem Solving in Primary Mathematics and Science Classrooms**

Presenter : Prof. Dr. Nerida F. Ellerton,Illinois State University, U.S.

Date & Time : 12 August 2003, 11.00 a.m. – 12.30 p.m.

1. Content of the Paper

1.1 Children learn through the medium of language. Language is an important element in the teaching and learning of science and mathematics. Sophisticated language skills are needed to develop sound problem posing and problem solving skills.

1.2 Mathematical and scientific communications make use of ordinary language. The language we use everyday is loaded with mathematical ad scientific terminology. The teacher plays an important role to assist learners to acquire the formal languages of mathematics and science.

1.3 There are two modes, receptive and expressive modes. In the receptive mode, students need to listen, to read, to observe, and to interpret. While in the expressive mode, they need to be able to express themselves to the teacher and peers verbally.

2. Discussion

2.1 Question

Ms. Teoh Sooi Kim of Sultan Abdul Halim Teacher Training College of Kedah Malaysia commented that with the change of instruction from Malay to English in the teaching of mathemtics and science, the teaching of these subjects has become very challenging. Very often, teachers have to translate from English to Malay and back to English. She enquired the presenter's opinion on double translation.

Answer

Translating the problem into Malay could be necessary in the beginning. It could be useful to explain what certain terminology means in Malay language, but teachers should guide students to solve the problem in English. To facilitate better understanding, teachers should link children's personal worlds in the teaching of mathematics and science. Children who are bilingual are at a better position. Children who have reached above the language threshold level would be most likely to have better performance.

2.2 *Prof. Yoshihiko Hashimoto of Yokohama National University, Japan* suggested that teachers should encourage students to pose problems and to use their minds to solve problems and visualize diagrams.

2.3 *Mr. Isa Othman of Limbungan Primary School, Malacca, Malaysia* emphasized that mathematics textbooks should be equipped with interesting mathematical ideas and extensive examples for students to have better understanding in the subject matter.