Do-Talk-Record – A Teaching Sequence for Developing Mathematical Thinking

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Abstract

Fostering mathematical thinking in learners is the central aim of all mathematics educators. What does this mean exactly? It means to infuse learners with an awareness of four mathematical processes – specialising, generalising, conjecturing and verifying, and to help these processes become alive and active inside learners. The more explicit teachers are about these processes in the classroom, the greater impact they will have on learners. The more explicit the learners are, the greater will be the power of their thinking. How can teachers help foster mathematical thinking in learners? Whilst the literature is rich with examples of activities that could help foster mathematical thinking, there is still the question of how to utilise these examples in mathematics classrooms so as to help learners develop mathematical thinking and to make mathematical thinking explicit and a part of learners' thinking repertoire. This paper, offers for discussion, a framework that utilises the teaching sequence do-talk-record and the processes specialising and generalising to help learners articulate their thinking and hence help make mathematical thinking more explicit in the mathematics classroom. This teaching sequence, underpinned by the psychological framework of manipulating-getting a sense of –articulating, is explored through three different but related problem solving activities. These activities and the framework were tried out with both pre-service and in-service primary mathematics teachers. The paper examines the importance of providing time for learners to see, experience and master the skills and concepts that were the objectives of the activities. Also the significance of the psychological framework that allows the teacher to track where learners were failing is discussed. The paper concludes that it is essential to allow learners to listen, discuss and do numerous examples of a pattern in order to generalise the pattern and then to apply the pattern to related problems to have a deeper understanding of the pattern.

Do-talk-record - a teaching sequence for developing mathematical thinking

Problem solving is an essential part of the Singapore mathematics curriculum (Figure 1). The curriculum recommends pupils be exposed to problem solving activities where they are encouraged to try and find different strategies to solve a given problem and "to create, formulate or extend problems" (CPDD, 2000, p. 18). In addition to problem solving tasks, it is recommended that pupils be exposed to investigative work whenever possible with the aim of engaging pupils with work that allows them to explore, experiment and discuss mathematical ideas. To support pupils in problem solving and investigative tasks, processes such as thinking skills and heuristics are to be infused into the mathematics curriculum.

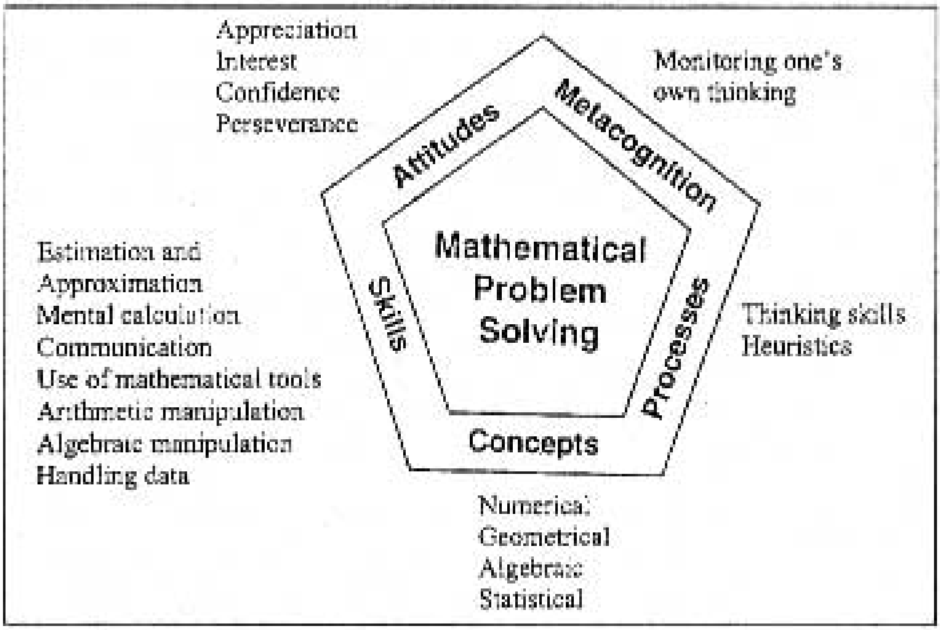


Figure 1: The Singapore Primary Mathematics Curriculum

Whilst the intended curriculum is well-structured, existing research literature has shown that often, the intended curriculum fails to take root as teachers responsible for implementing the intended curriculum continue to practice what they are already familiar with. While terms like problem solving, investigations, thinking skills and heuristics have been part of the research literature for more than two decades, teachers implementing the curriculum may not be familiar with them and hence are not confident as how to conduct such activities. The beliefs literature (e.g. Thompson, 1984, Ball, 1988, Cooney, 1985) has findings to show that while teachers often espoused the intention to implement the new initiatives into their lessons, in reality they fail to do so as they themselves have not been socialised with such mathematical activities and their pedagogical knowledge does not have a framework for conducting such activities in their classrooms. (See Alba Thompson's (1992) Teachers' Beliefs and Conceptions: A synthesis of the research for a fuller discussion on this area.) Problem solving and investigative tasks as defined by current research literature are not the standard fare of practicing teachers who were in school 20 or more years ago. Therefore, based on the findings of beliefs literature, it is quite logical to state that practicing teachers would face difficulties developing such activities in their primary classrooms. Anecdotal evidence collected through work with both in-service and pre-service teachers suggest that this is indeed the case.

How can teachers help foster mathematical thinking in learners? Whilst the literature is rich with examples of activities that could help foster mathematical thinking, there is still the question of how to utilise these examples in mathematics classrooms so as to help learners develop mathematical thinking and to make mathematical thinking explicit and a part of learners' thinking repertoire. This paper, offers for discussion, a framework that utilises the teaching sequence do-talk-record and the processes specialising and generalising to help primary mathematics teachers articulate their thinking and hence help make mathematical thinking more explicit in the mathematics classroom. This teaching strategy and the related psychological framework are offered by the Open University course EM 235 Developing Mathematical Thinking and further developed by Barbara Jaworski in her book and in the classic book *Thinking Mathematically* by John Mason, Leone Burton and Kaye Stacey (1982). Mason (1996) gave a detailed account of the psychological model in the article Expressing Generality and Roots of Algebra.

In section one, the significance of each phase of the psychological framework is explored through three related mathematical investigations. How this psychological framework is used to inform the teaching strategy do-talk-record is discussed in section two. This discussion is based on the responses of in-service teachers who are engaged in the investigation activities. This section also examines the importance of providing time for learners to see, experience and master the skills and concepts that were the objectives of these activities. The paper concludes that it is essential to allow learners to listen, discuss and do numerous examples of a pattern in order to generalise the pattern and then to apply the pattern to related problems to have a deeper understanding of the underlying structure of the pattern. When new initiatives are introduced into the mathematics curriculum, mathematics teachers require time to engage with these initiatives in a way that would support them and give them confidence to implement these initiatives.

Manipulating – getting a sense of – articulating – the helix underpinning the do-talk-record teaching strategy

The psychological framework,, in the form of a helix, consists of three learning phases 'manipulating – getting a sense of – articulating' is based on Bruner's – enactive-iconic symbolic - model of learning. The helical framework loops round and round upwards with each loop representing an ever deepening understanding of a particular concept (Mason, 1996). Each loop of the helix represents the three learning phases learners encounter when they engage with a particular mathematical object. The mathematical object includes concrete objects, symbols or abstract ideas. First, learners engage with the mathematical objects by manipulating and exploring them and trying to see the pattern or the underlying structure. After getting a sense of the pattern or the structure, the learners can then generalise the pattern by articulating the pattern either orally or in written form.

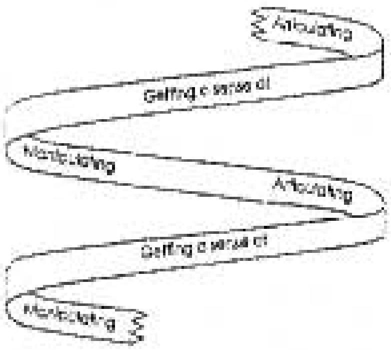


Figure 2: The helix

An example is the case when children learn about the shape triangle. The children work with the shapes provided by the teacher. The children manipulate these examples of triangles and non-triangles and identify those that are triangles. These children then acquire a sense of the shapes called triangles and gain confidence working with such shapes. The children articulate the generalised idea of triangle when they are able to find their own examples of triangles. Another example is when children work with numbers. The teacher teaches the children about the number one. Examples of oneness is shown to the children and the children manipulate with the sets of one objects provided by the teacher. With more examples of sets of one, the children get a sense of one when the children detect what is the same of sets of one and how sets of one are different from sets of not one. Once children get a sense of oneness and are confident and comfortable with the concept of oneness, these children then pick their own particular examples of sets of one from the environment. At this stage of articulation, these children are said to have generalised the concept of oneness and thus are able to articulate the idea of oneness. Articulation need not to be verbal; rather it could be concrete, diagrammatic or symbolic. The pattern achieved in the articulation phase becomes available for manipulation in the next loop of the helix. In the example with the triangles, the children can use their knowledge of triangles to identify specific types of triangles within the general idea of triangles. The children could differentiate and specialise triangles that are isosceles from those that are not. In the case of number one, the children, at the next level, learn to add one more other object and generalise about twoness. Should the children become unclear with the pattern generalised at the first loop, they can retrace down the loop and review the articulations for further clarifications.

The Activities with the In-service Teachers

The helical model can be used to analyse how mathematical thinking can be developed and in this paper I will explore this model through three activities. These activities were chosen as part of an investigation – problem solving exercise for in-service teachers (See Appendix 1 for the course material presented to the teachers.) The in-service teachers signed up for the Module 6 – Enrichment Activities for Primary Mathematics either as part of an Advanced Diploma course or as a standalone module. This is a twelve week three-hour module conducted by the National Institute of Education. Throughout this module, teachers' attention was drawn to the concrete-pictorial-abstract approach to teaching mathematics advocated in the curriculum.

The teachers took two sessions to work through these three activities. All the participants were seated in groups of four, two pairs in each group. However all teachers had first to work individually on each activity. Once they have established certain patterns in the numbers, they then checked their observations with their partners, and then each with the other pair in the group. Once the entire class had completed this stage the teachers were asked to share their findings with the entire class. I used the strategy where one group reported one finding which was then recorded on the white board. The next group then reported another finding different from the first group and their finding was then recorded on the board. This process was repeated with each group until all the findings were listed on the board. When a particular feature was not noticed by the teachers, a prompt in the form of the question was provided to the class. Teachers' responses to these prompts were then recorded on the board.

Table 1: In-service teachers

In-service teachers	
Standalone	Advanced Diploma
15	13

It was not possible to audiotape the discussions held by the teachers as there were seven groups. Also the talk of the teachers made it impossible to listen to and transcribe the recording. Copious notes of the two lessons were made at the end of each lesson.

Do-Talk-Record Teaching Sequence and its Relation to the Mathematical Processes
This teaching sequence was first introduced by Nick James (Floyd, 1981) a member of the EM235 course team. In this section, the teaching sequence is discussed in relation to the mathematical processes emphasised in the helix.

Doing and Specialising
Here learners manipulate with objects, either concrete or abstract, until they are confident of the materials. Learners manipulate with several examples of the mathematical object until they get a sense of the pattern or underlying structure of the mathematics embedded in the doing. At this stage of the learning sequence, learners utilise previously mastered knowledge or skill to help them get a grasp of the ideas or mathematical structure embedded in the set task. For example, in the case of the Triangular Number Activity,

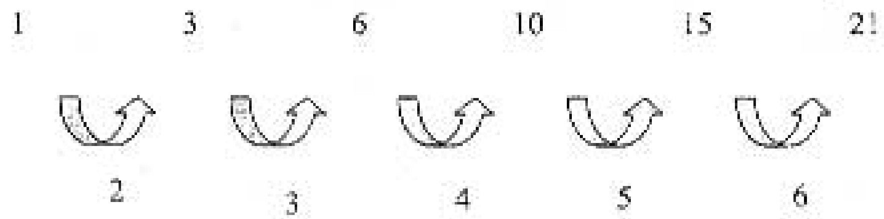
teachers utilise their knowledge of triangles to investigate the properties of triangular numbers. Each teacher was given a set of round counters. They were asked to build the structures listed in the handout. After manipulating the counters and forming figures 1 to 4, the following findings were offered by the teachers.

- The numbers are called triangular numbers because each set of counters formed a triangle.
- To form each consecutive figure, you had to add one more row of counters and the number of counters added is the same as the figure number.
- The figure number matched the number of counters in the bottom row.
- There is one dot in Figure 1, three dots Figure 2, six dots in Figure 3 and ten dots in Figure 4.
- There were as many counters in the bottom row as there were going up. For example in Figure 4, the triangle had a base of four counters and a height of four counters as well.
- The triangles were equilateral triangles.
- Figure 5 has 15 dots and Figure 6 has 21 dots.

While the teachers were able to state how many dots there were in Figures 5 and 6 by using the recursive rule of adding the figure number to the previous triangular number, no attempts were made at generalising the pattern they see. So the prompt was "Write out the dots in each Figure as a sequence. Describe what you see?" This pattern was offered by the teachers: Figure 4 is 1 + 2 + 3 + 4 and Figure 5 is 1 + 5 + 3 + 4 + 5. Consequently the teachers were able to state the number of dots in Figure 5 and Figure 6. To help the teachers at generalising they were asked to comment on what the sequence for Figure 10 and Figure 20 could be. Because they could see the sense of the pattern structure for the first five figures, they had no difficulties extending the pattern to the 10th figure, the 20th figure and finally the nth triangular number.

Talking, Recording and Generalising

At this stage the teachers talked about their specialising and they tried to capture the underlying structure of the mathematical idea they were engaged in. Once the teachers had articulated the number of dots in each figure, they were ready to relinquish the concrete representation of triangular numbers and formed the triangular number sequence of



by constructing and recording the recursive rule of $T_{n+1} = T_n + n$. However teachers had difficulties when they were asked to express, in conventional language, the rule to generate the next triangular number. In particular the teacher had difficulties explaining what n was. This was then a good point in the investigation for the teachers to return to their specialising phase and re-examine the relationship of n to the concrete triangles. By re-examining the structures and the symbolic representation for the number sequence, the teachers were able to explain that the letter n referred to the figure number or the position number of the triangle and hence the number. For example to get the fifth number, the teachers knew that they had to add 5 (the fifth triangular number) to 10 (the fourth triangular number). Hence although the teachers were able to express the next triangular number from the given sequence they could only make meaning of the pattern by returning to the concrete representation and linking the concrete representation with the symbolic form. They also could not tell how many dots there were in the 10^{th} and 20^{th} figure as it was not reasonable to make such figure. Surely there has to be a short cut!

In the above discussion, the fact that the teachers had to talk and then record their patterns in symbolic form helped them to crystallise their thoughts further as they had to use both the concrete and pictorial forms to check that their symbolic forms were meaningful.

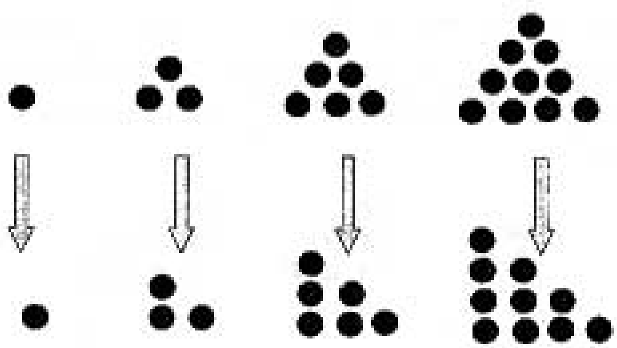


Figure 1 Figure 2 Figure 3 Figure 4

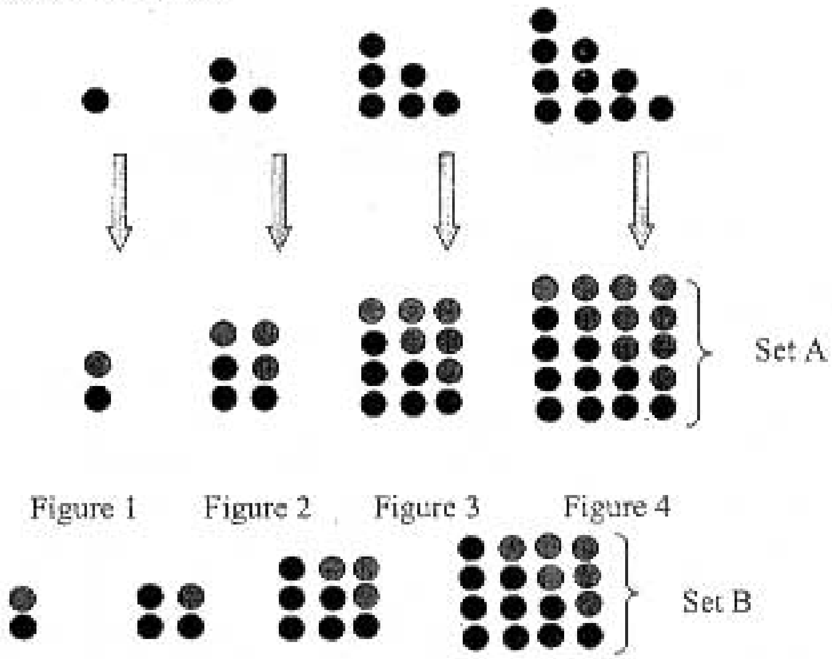
Doing and specialising - again

How many dots were there in Figure 5? Figure 6? Figure 10? Figure 20? Teachers were able to use the recursive formula to work out the total number of dots in Figures 5 and 6 When asked, the teachers knew that in Figure 10, there were 1 + 2 + 3 ...... + 10 dots and 1 + 2 + 3 + ... 20 dots in Figure 20. Once they had a sense of the underlying structure they were able to state the number of dots in Figure 50 and so on. By this process of doing and specialising, teachers were able to identify the number sequence for each triangular number. However they were unable to work out the sum of each sequence. Although some of the teachers were able to give Gauss's formula for the sum of first n natural numbers for Figure 10 as $(1 + 10)\frac{10}{2}$ they were unable to explain why the answer should have this particular structure. These teachers had learned the formula from the handshake activity which they encountered in some previous activity.

The teachers were then asked to consider the structure of triangular numbers. How could they use the physical structure of the triangle and the counters to help them work out the total number of dots in each figure and hence the sum of each sequence? Because such activities are alien to teachers, teachers needed guidance as to how they could use the concrete counters to help them answer the question. Also because they had identified the triangles as equilateral triangles, the teachers seemed to think that they had to maintain the equilateral structure. The teachers were asked if they could re-structure each of the equilateral triangles into triangle without adding or removing any of the counters from each triangle. The teachers found this question rather challenging. Teachers now worked in their groups, and after some manipulation with the counters, they were able to construct the corresponding right-angled triangles. This activity broadened their insight into the structure of triangular numbers – the triangles need not be equilateral.

Manipulating the counters into the corresponding right-angled triangles gave the teachers some inkling as to how they could use the areas of triangles to work out the total number of counters in each figure. They though the area of each respective triangle would give the total number of dots, as long as they increased the height of each triangle by one. The base of each triangle corresponded to the figure number.

To help teachers move out from this 'struck' position, they were asked if they could complete the triangles by constructing the corresponding rectangle for each triangle. The completed rectangle had to maintain the structure of the original triangle. A possible completion looked like Set A.



However some teachers had completed their triangles as in Set B.

The teachers were asked to compare the two sets of rectangles and to provide reasons which set was more likely to lead to them to find the total number of dots in a given figure. At first teachers were puzzled how the rectangles could provide the solution to the number of dots in each triangle. The teachers were asked to re-consider the relationship between the area of rectangles and their related triangles. With this prompt the teachers began to consider the appropriateness of each set of rectangles. The discussions of the teachers working in their groups showed them making slow progress. They reasoned that the area of triangles based on the Set A rectangles gave the solution to the total number of dots of related triangular numbers. Although a verbal formulation began to crystallise, none of the teachers saw the need to record their findings. Because such tasks were new to teachers, the teachers were asked to make a table and to record their findings in such a was as to help them construct a rule for finding the number of dots in Figures 10 and 20. A discussion was held among the teachers to decide how to record their finding. I proffered to the teachers that their question would be better answered if they evaluated what they knew (I know) and what they wanted to find out (I want). One teacher suggested making a systematic list which contained the different triangular numbers. After much debate the following table (Table 2) was constructed. This table enabled them to check their verbal formulations against the figure number, pictorial representation of each figure and the number of dots in each figure.

Table 2

Figure Number	Number of dots	Triangle and its related rectangle	Area of rectangle	Area of triangle	Number of dots in each figure
1st	1		1 x 2	$\frac{1}{2}$x (1 x 2)	1
2nd	3		2 x 3	$\frac{1}{2}$x (2 x 3)	3
6th	?		6 x 7	$\frac{1}{2}$x (6 x 7)	21

and the intermediate numbers

any figure number	?			$\frac{1}{2}$ (any figure number x any figure number + 1)	

Only when the verbal formulations were recorded did the teachers see the connections between the figure number, the total number of dots and the area of the triangle. Teachers were encouraged to check and re-test their formulations with the concrete constructions and the corresponding pictorial representations. It was not sufficient that the teachers came up with the formula but it was important that teachers see what they need to do once they had a generalisation. They had to re-visit and check their generalisation with the objects. By having to record their verbal formulations, teachers were forced to stand back and re-look at the problem with a fresh perspective.

The above investigation took about two hours to complete. The lesson ended with the teachers recording their verbal formulation as generalisation for finding the number of dots in the n^{th} figure. At the end of the lesson, the teachers had some ideas of triangular numbers. For the teachers to test their understanding and for their understanding to take hold, they needed further practice so that they could manipulate the idea of triangular number for further specialisation and generalisation. At the next in-service session, the teachers were then presented with the Crossroad and All Shook up problem.

Crossroad Problem

Each teacher was given straws to model the crossroad problem. Because of their previous experience with the triangular numbers, they were more confident as to what to do with the straws. They were more systematic with the modelling, starting their exploration for the number of crossings with two roads, three roads and so on. They also recorded their findings in a systematic list. The table given in the handout helped focussed their thoughts as to how to record their findings. The teachers had no difficulty answering the questions posed in the worksheet as well as coming up with a generalisation for the number of crossroads for any number of roads crossing. The teachers expressed surprised that the solution of this problem were triangular numbers. Hence it is important that investigations be extended so that learners are made aware that an investigation is not just a once off activity to be completed and forgotten. Such awareness might make them look for mathematical patterns that have the same structure as triangular numbers.

All Shook up

The teachers were asked how they could get a sense of this problem which is the least structured of all the three. Although the heuristic 'acting it out' was offered, it was not clear how to act it out and how this could help the teachers make sense of the problem. A discussion ensued looking at what would happen at a party if only one person was there. How many handshakes would this one person exchange? From this the teachers concluded that the least number of people at the party has to be two for this problem to have a meaning. Two teachers came out and acted out 'one' handshake. A third teacher came out and they shook hands and the teachers counted two handshakes. The process was stopped at the third teacher as I wanted the teachers to re-evaluate how this modus operandi would help them get a sense of the problem. Was it necessary to count the handshakes with eight people at the party? Would they do the same if the question asked for 20 people? If a pattern emerged would they be able to see it? With these prompts, the teachers concluded that they should record their 'acting out'. What form should these recording take? The following table (Table 3) was finally agreed upon.

Table 3

Number of people	Number of handshakes exchanged	The total of handshakes exchanged
1	0	0
2	1	1
3	1 + 2	3
4	1 + 2 + 3	6
5	1 + 2 + 3 + 4	10
8	1 + 2 + 3 + 4 + 5 + 6 + 7	

To make better sense of the problem, the 'acting out' was slowed down to address the question "What happens when the fourth person comes to the party? How many handshakes will this person exchange with the others already there?" Because the process was slowed down, the teachers saw that the fourth person only needed to shake hands with the three people already there. The recording then added '3' to '1 + 2'. Because the 'acting out' process was slowed down the teachers were able to get a sense of the problem and the pattern as to how the handshakes for 5 people would be 1 + 2 + 3 + 4. From here the teachers could generalise the number of handshakes exchanged among 8 people. One teacher said 'Oh triangular numbers". The total number of handshakes could then be easily worked out. However some of the teachers were confused because they directly applied the first generalisation for working out the nth triangular number as $\frac{n(n+1)}{2}$ to the handshake problem. Rather than taking the total handshakes for 8 people as the 7[th] triangular number, the teachers took it as the 8[th] triangular number and erroneously calculated the total number of handshakes as $\frac{8 \times 9}{2}$. Therefore while these teachers manipulated the generalisation achieved at the lower stage, they needed to get a sense of the existing problem and see how the solution achieved at the lower can be manipulated to help them generalise further for another related but not identical problem.

Conclusion

These three activities took a total of five hours to complete. Even at the end of the second session, some teachers wanted more time with the 'All Shook Up' problem as they felt they needed to mull over the generalisation for total number of handshakes for any number of people. All through the three activities the teachers were given time to engage, on an individual as well as on a group basis, with concrete materials in the form of counters, straws and people, the pictorial as well as the the symbolic representations of the patterns they see. They were encouraged to describe in their own words what they see. The teachers were also comfortable with the idea that a wrong response was not an offence but often wrong responses helped generate more discussions, hence further clarifications. They realised the importance of concrete materials as this helped them to think through the problem. It was interesting to note that while the Singapore primary mathematics curriculum advocate the meaningful development of concepts and skills through the phases concrete-pictorial-abstract, teachers were often unsure what use to make of concrete materials. This evidence lends credence to the findings reported in beliefs literature that the introduction of initiative must be supported by suitable in-service courses that will enable teachers to experience and see how these new initiatives are actualised in classrooms. Without such experiential support, teachers have no notion how these new initiatives can be actualised and how learning takes place, and hence are unlikely to conduct well intended activities with their pupils.

The do-talk-record teaching sequence was an effective framework that helped me work with the teachers. Also the teachers began to have a sense of how the lesson would proceed and what they need to do to help their thinking become more explicit. Further research needs to be conducted to see how teacher can apply the do-talk-record sequence in their classroom teaching.

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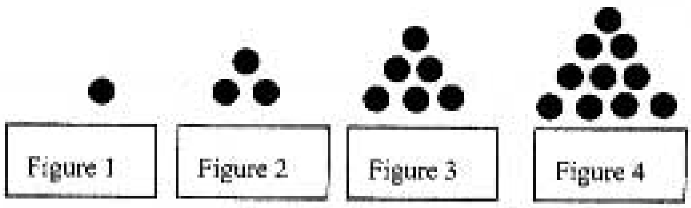
Appendix 1

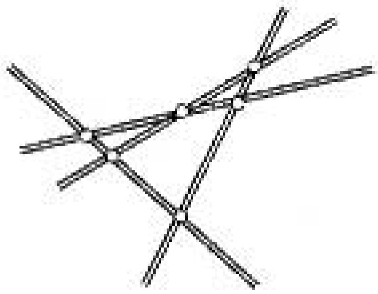
Triangular Numbers

Look at the series of dots in Diagram 1.



1. Describe how you would build Figure 2 from Figure 1, Figure 3 from Figure 2, Figure 3 from Figure 4.
2. Record the number of dots in each figure. How many dots are there in figure 5? Figure 6? Figure 10? How many in Figure 20?
3. Is there a shortcut to help count all the dots in the Figure 10, Figure 20 without having to count all of them? State the shortcut in words

How many crossing points are there for these crossroads?



Draw a fifth road that crosses each of the four roads.
How many crossing paths are there now?
You can show the number of crossing points as dot patterns.

Number of roads	Number of crossings	Dot patterns

How many crossings would you expect there to be for 10 roads?
How many crossings would you expect there to be for 20 roads?
(Source: Kerslake et al, y4)

All Shook up
Eight people meet at a party. They all exchange handshakes. How many handshakes are exchanged? Extend this problem. How many handshakes would be exchanged if there were 10 people? 20 people? Is there a shortcut to help count all the handshakes? Provide alternative solutions to the problem.

Answer
Grade 5

2.2 *Mr. Norjoharuddeen Mohd. Nor of SEAMEO Regional Center for Mathematics and Science, Penang, Malaysia* wanted to know how these Do-Talk-Record activities are documented in the Singapore Primary Mathematics Curriculum.

Answer
These activities are outlined in the curriculum guide and further enhanced in the textbook with a list of specific activities to be carried out in the classrooms.

2.3 *Teoh Sooi Kim of Malaysia* enquired the ways we could encourage our teachers to carry out these Do-Talk-Record activities through modeling behaviour.

Answer:
This can be done through in-service programs whereby teachers gather weekly and try out the processes with the group before implementing it in their classrooms. They will then come back to the group to discuss and exchange their experiences.

Identifying and Remediating Pupils' Misconceptions On Electricity

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Abstract

A wide range of techniques can be used to explore pupils' thinking about science concepts. This paper summarizes a number of ways teachers could use to provide information on pupils' misconceptions. We define misconceptions as strongly held cognitive structures that are different from the accepted understanding held by scientists in a field and that are presumed to interfere with the acquisition of new knowledge. One effective way to probe children's understanding and misconception of science concepts is the Prediction-Observation-Explanation (POE) activities. It is the aim of this paper to demonstrate a few POE activities that could identify pupils' misconceptions on electricity. This paper also briefly review the common misconceptions held by pupils on electricity as found in previous studies. Finally this paper provides some suggestions on ways to remediate pupils' misconceptions of basic electricity. Some of the activities that teachers could use on their own pupils are analogies, experiments that lead to conceptual change and active discussion activities.

Identifying and Remediating Pupils' Misconceptions on Electricity

1.0 Introduction

There has been an extensive research on children's prior ideas about natural phenomena that they bring to science classes. When prior ideas are inconsistent with the accepted scientific ideas, these ideas are considered as misconceptions. Some of these misconceptions are said to be highly resistant to change even after receiving formal instruction. This is because teachers tend to assume understanding has been achieved after a typical formal instruction where concepts are carefully explained or an experiment has been done on the concepts. Research on pupils' misconceptions suggests that for an effective science teaching, teachers should take into account pupils' ideas and help them by providing activities that enable pupils to change from their current understanding to a more scientific view.

Therefore it is important that teachers given the opportunity to probe pupils' understanding and in the process are able to identify pupils' misconceptions. By using techniques to identify pupils' misconceptions, teachers will appreciate pupils' understanding and misunderstanding, they will then teach more interestingly and student will find learning better (White and Gunstone, 1992).

It is the aim of this paper first to describe a summary of the various techniques that could elicit pupils' misconceptions. This is followed by a brief summary of previous studies and findings on pupils' misconceptions of a particular physics concept namely electricity. Second this paper provides a few activities to identify pupils' conception of basic electricity, through the Predict-Observe-Explanation technique. Finally, this paper will also provide some suggestions on ways to remediate pupils' misconceptions of electricity.

1.1 Techniques of probing pupils' misconceptions

A wide range of techniques can be used to explore and probe pupils' thinking about science. The following is a list of techniques which is not meant to be exhaustive but they have been used in science research and have been successfully trailed in science classes (Driver, Squires, Rushworth and Wood-Robinson, 1994 ; White and Gunstone, 1992). For each of the technique, a brief description of it and the procedures for implementing it are discussed.

a. Interview

The interview is one of the oldest techniques used to probe pupils' thinking. It is the most direct method of assessing pupils' understanding of concepts. Through interviews, we are able to gather as much information as possible about what a pupil knows about a concept. A teacher must remember that the purpose of the interview is not to find out only correct answers but the reasoning they use to reach the answers. By careful probing, a teacher is able to uncover how pupils reach correct conclusions based on false premises. This demonstrates how correct answers may sometimes conceal the presence of misconceptions.

Procedurally, the interview should allow the pupils to respond freely on his or her own accord to the questions and the manner in which the thought is unfolded. In addition, teachers also need to probe the reasoning they have used to answer the question. It is important to do this whether the pupil gave a correct or wrong answer. Hence, a teacher needs to respond to any answer in a neutral way. To explore the reasoning further follow up with a neutral probe such as 'Why did you say that?' Keep your voice tone encouraging without displaying any indication of whether the response given was right or wrong. Here are some possible interview questions for the topic 'Solution'.

- What happens if you put some salt in water and stir it? How does this happen?
- Is the salt still there?
- What happens to the weight of the salt?
- What happens to the weight of the water after the salt is added?

(Probing questions would include: 'Why did you say that?' or 'Can you explain why?' or 'Can you tell me more about that?')

The interview technique, when used in a classroom environment is similar to the question and answer session between a teacher and his pupils. The current Q&A session conducted in the classroom, however, is more teacher-initiated and students respond to the questions from the teacher. The question initiated often has only one solution and the need to probe is unnecessary. White and Gunstone (1992) advocated promoting question asking by students as it tends to reveal more of their understanding than misunderstanding.

b. Concept mapping

Concept mapping is a technique which requires the pupils to define the relationships between concepts or ideas. Concept mapping can be used as a precursor to interviews and also as an end-product. White and Gunstone (1992) and Monk and Dillon (1995) outlined the steps pupils should go through in producing a concept map.

a. Each concept to be mapped is written on small pieces of paper (3cm x 3cm). This is to enable pupils to manipulate physically the concept labels when they are thinking about their concept maps.
b. Pupils are to sort out the concept labels into two piles - those they know the meaning and those they do not understand.
c. Pupils are to lay out the remaining cards on a large sheet and arrange them in a way that makes sense to them.
d. When the pupils are happy with the arrangement, stick the labels to the sheet.
e. Pupils are to draw lines between the concepts that are related or connected.
f. Pupils then add a few words to the lines to explain how they are connected.
g. The pupils are to go back to the 'do not know' pile and see if they can add anything to the map. If they add to the map, make sure they write the nature of the links between them and also the other terms.

Figure 1 is an example of a complete concept map of 'Living Things' that encompasses the following: Living things, Animals, Cow, Monkey, Banana tree, Plants and Grass.

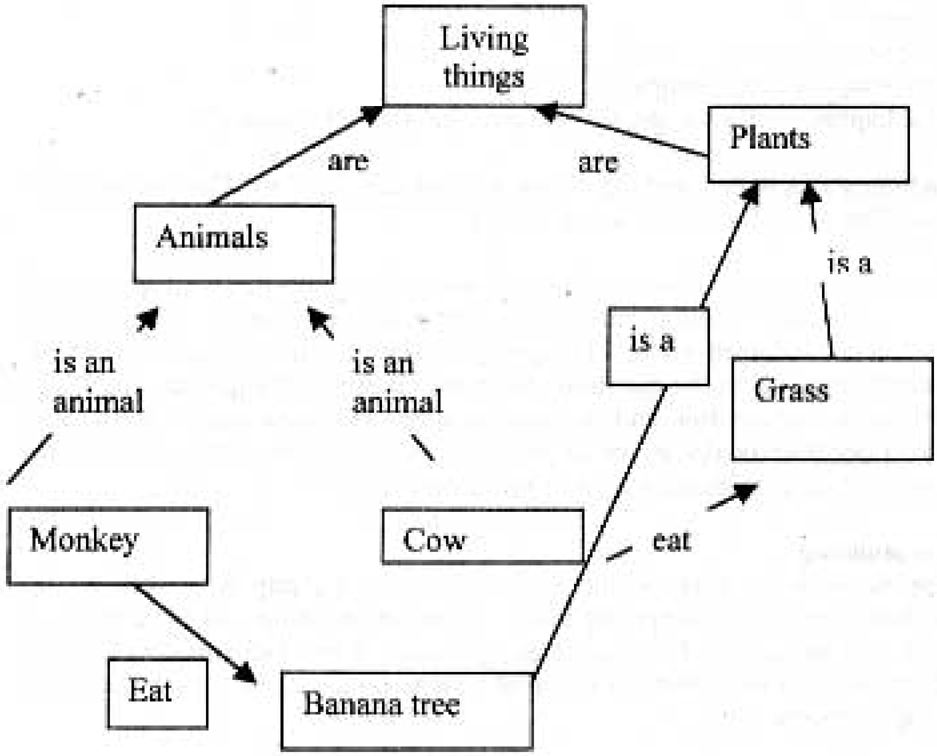


Figure 1: A complete concept map

Figure 2 is another example of a concept map. This concept map suggests that the pupil has a low level of understanding of electricity since it has few links and the connection between electrons and static is questionable (White and Gunstone, 1992).

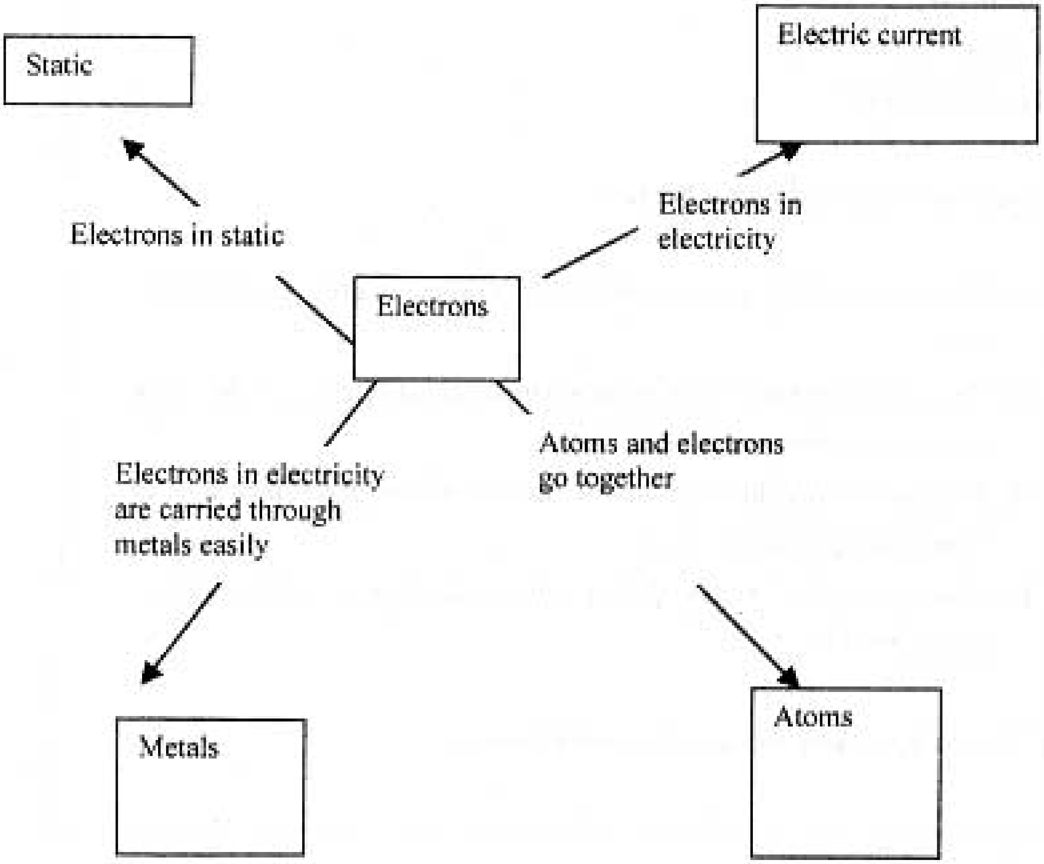


Figure 2: Concept map indicating a low level of understanding

c. ***Multiple choice tests***

Multiple choice tests have been used to identify pupils' misconceptions. This technique allows the teachers to easily identify misconceptions held by a group of pupils by administering a pencil and paper multiple choice test which has items designed to identify misconceptions. The multiple choice test items are innovative in that the distractors for the items are based on findings of pupils' misconceptions of the concepts. Often, the test consists of two parts. The first part is a multiple choice content question with two or three choices. The second part of each question contains a set of four possible reasons for the answer given in the first part. The reasons consist of a correct answer, any identified misconceptions and a wrong answer. Such a format could not only detect misconceptions but it also prevents pupils from guessing the answer for the first tier of the questionnaire. An example of an item from a two-tier diagnostic test is shown in Figure 3.

Which gas is taken in by green plants in large amounts, when there is no light at all?

(1) carbon dioxide gas

(2) oxygen gas*

The reason for my answer is because:

a) This gas is used in photosynthesis which occurs in green plants all the time.

b) This gas is used in photosynthesis which occurs in green plants when there is no light energy at all.

c) This gas is used in respiration which occurs in green plants when there is no light energy at all.

d) This gas is used in respiration which takes place continuously in green plants.*

* This is the correct choice and correct response

Figure 3 : An example of a two-tier diagnostic test

2. ***Predict-Observe-Explanation (POE)***

This technique probes pupils' thinking by asking them to do three tasks. First, they must predict the outcome of some event, and must explain their predictions. When pupils are to give predictions to a phenomenon, they would need to apply their knowledge of science that they believe to be most important. Sometimes, the pupils' reasoning might be based on their everyday experiences and thus the POE technique as argued by White and Gunstone (1992) has the greatest power compared to any other techniques in probing pupils' prior ideas which they use to interpret real events. The second step in the POE technique is that the pupils then try them out and describe what they see; and finally they must reconcile any conflict between prediction and observation.

Below is an example of response to a POE task on the topic of air resistance:

> Two balls of the same volume but in different weights were released from the same height. Pupils were then asked to predict which ball would reach the floor first and give a reason. The reasons given were often based on everyday experiences which were inconsistent with the scientific view such as 'the heavy ball reached first', 'it faced less resistance' and 'gravity pulled equally hard on all objects'. When they observed the experiment they found out that the two balls reached the ground nearly at the same time. After the observation, the pupils would need to reconcile their prediction with the observation by giving such explanation as 'the same size hence experienced the same air resistance'.

In doing POE, White and Gunstone (1992) suggest that teachers must ensure the following steps are taken so that the POE task is effective in identifying pupils' misconceptions and thus guide their understanding of the concept being discussed. Firstly in choosing the task, one must ensure that the problem presented is real to the students. For example, the above POE tasks used balls rather than pendulum bobs. It is also important that each pupil indicates both his or her prediction and give reasons. Making a prediction without giving reasons would defeat the purpose of POE which is to probe pupils' understanding and misconceptions. Pupils need to be trained to write down their observations. If they fail to do that, then they can write their observations after hearing from the others. In this instance, their own observations could be different from the rest of the class. Pupils need to be encouraged to suggest a plausible explanation to reconcile their prediction and observation so that through the new explanations given, pupils can understand better and this may overcome their misconceptions.

1.3 Misconceptions on electricity

Various studies have shown that children may have a number of misconceptions on electricity which are related to basic concepts such as current flow (Osborne and Freyberg, 1985, and Shipstone, 1984), and voltage (Psillos, Koumaras, Tiberghien, 1988). Psillos et al. (1988) found that even though pupils were familiar with basic symbols of electrical circuits, there was confusion between voltage and power. They also noted that there was a tendency for pupils to 'equate resistance with the amount of materials', that is, a short wire had lesser resistance.

Osborne (1981) conducted a number of studies on pupils' ideas of electric current; on how current flows and on what happens when electric current goes into a lamp. He found that pupils held four different models on how an electric current flows in a simple circuit consisting of a battery, bulb and wires. The models are the 'unipolar model', the 'clashing current model', the 'consumed (weakening) current model' and the 'scientific model' as shown in Figure 4.

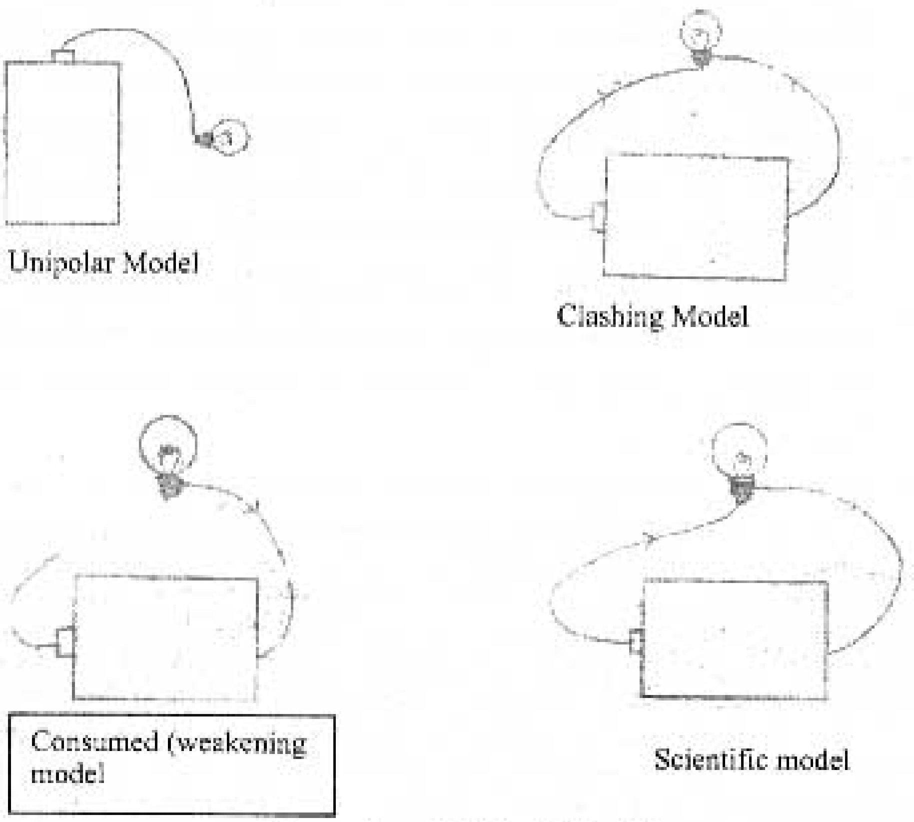


Figure 4 Models of Electricity

Source: Driver et al. (1994). Making sense of secondary science teaching

In the unipolar model, pupils regard only one wire is needed for a complete circuit. In the clashing model, pupils think that current flows from both terminals of the battery to the bulb. In the consumed model, pupils think that the current is being used up by the bulb and therefore it goes back to the battery. Lastly, in the scientific model pupils argue that the amount of current is unchanged as it travels around the circuit. Osborne (1981) believes that electric current is the most important and basic idea and advocates that pupils should first understand about electric current as a pre-requisite to an understanding of prerequisite the other aspects of direct-current circuits.

Studies by Shiptone (1985) and others went further than the concept of current flow. They studied the concepts of resistance and voltage and the effects of resistance on the distribution of current. Shiptone found that a vast majority of pupils held a 'sequence model' of current flow, and he regarded this misconception as representing 'a fundamental misunderstanding of the behaviour of circuits'. This error implied a model of current flow in which the current, as it progresses around the circuit is influenced by each element that it encounters in turn. Thus, pupils would consider the position of a resistor before or after a lamp in a simple circuit has an effect on the brightness of the lamp.

Children also experienced difficulty in recognizing elements of a circuit diagram in a practical situation using real equipment. According to Grayson (1997), circuit diagrams allow us to represent a circuit on a paper by using symbols for the batteries and bulbs instead of drawing pictures. The symbol for a battery –||– does show it has two different terminals. The symbol for a bulb, however, shows no difference between the two terminals. Also, the circuit diagram tells us which points are electrically connected, but does not give a detailed information about the physical layout, e.g. whether the circuit elements are touching directly or connected by a wire. Hence, it is advisable that teachers teach the students to connect the real equipment first before introducing them to the circuit diagrams or teachers can do the physical and abstract set-up concurrently.

1.4 POE activities to identify misconceptions of electricity

In this paper, we will only focus on two well-documented misconceptions of electricity: a) models of electricity and b) the idea that a battery is a source of constant current.

a. *Models of electricity*

Baker and Piburn (1997) argue that the activity which quickly reveals preconception or misconceptions about electric circuits is when asking them to light one light bulb with a battery and a piece of wire. This activity also highlights the need for pupils to observe the similarities and differences between symbols and the actual physical appearance of the circuit elements such as bulbs and batteries. The procedures for this activity are as follows:

1. Suppose you are given the following items: a battery, a light bulb and a wire.

2. Draw two diagrams – one that will make the bulb light up, and two, that will not make the bulb light up.
3. Make sure you draw the parts of the bulb and battery clearly.
4. Next to the diagram explain why you think the bulb will or will not light up.
5. Then connect the battery, bulb and wire together as you have drawn and predicted.
6. Write down what you observe.
7. Give the reasons why the set up did or did not work.
8. State the requirements that must be met in order for the bulb to light.

Figure 5 (taken from Baker & Piburn (1997)) shows the various arrangements that will or will not light up the bulb using one wire.

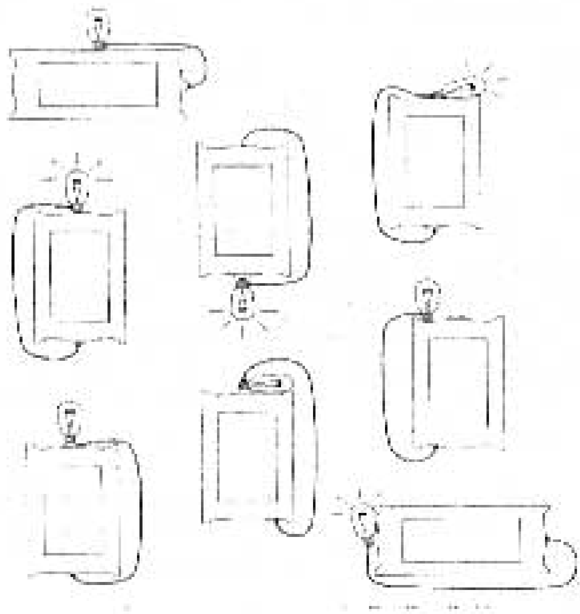
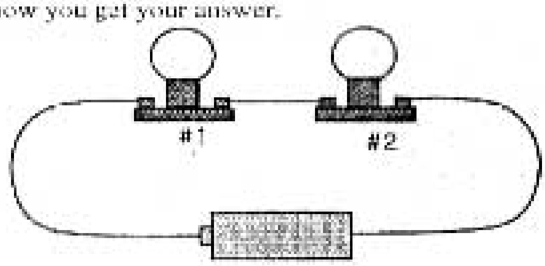


Figure 5: Arrangements that will or will not light up the bulbs

Is battery a constant current source?

This activity is useful to test or identify the misconception of pupils that the current is used up and a battery is a constant source of current. The set-up and instructions are shown in Figure 6.

1. For the circuit shown below, predict how bright you think bulb #1 will be compared to bulb #2. Explain how you get your answer.



2. How do the brightnesses of bulbs #1 and #2 in the circuit above compare to the brightness of the bulb A in the circuit below? Explain how you get your answer.

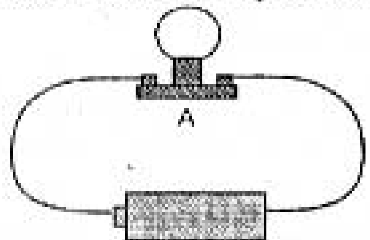


Figure 6 : Circuit Set-up and Instructions

Figure 6 is taken from Grayson (1997) in a workshop on approaches to helping physics teachers develop different types of knowledge. The procedures are given below for the observation and explanation tasks. The first two steps are shown in Figure 6.

Step 3

- Set up the circuit with the two bulbs connected as shown in step 1 of Figure 6. Compare the brightness of the two bulbs to each other and answer the following questions:
- Are your observations the same as your predictions?
- What can you conclude from this observation about the amount of current through each bulb?
- Is the current "used up" in the first bulb, or is the amount of the flow the same through both bulbs?
- Do you think the order of the bulbs in this circuit might make a difference? Verify your answer by switching the two bulbs.

Step 4

- Set up the single bulb circuit alongside the two-bulb circuit
- Compare the brightness of each bulb. Compare your observation with your predictions
- Based on your reasoning, what can you say about the amount of current flowing through a single bulb circuit with the current through the same bulb when connected in series?

The answer to step 3 is that both bulbs will light up with the same brightness regardless of its position. This indicates that current is not used up as it flows through the circuit. While in step 4, the brightness of Bulb A is brighter compared to Bulbs 1 and 2. If we assume that brightness of a bulb is equivalent to the amount of current than we can deduce that the amount of current, in circuit 2 is higher than circuit 1. Grayson (1997) explained that a light bulb is an example of a circuit element that presents an obstacle to the flow of current. Adding the number of bulbs mean increasing the resistance of the circuit. When the resistance increases, the current in a circuit decreases.

1.5 Suggestions to remediate misconceptions of basic electricity

As indicated above, the POE tasks are able to identify misconceptions and also to challenge and develop a sense of dissatisfaction of the pupils' ideas so that they are forced to reconsider their earlier preconceptions and perhaps change towards the scientific view.

Grayson (1997) suggests that where possible teachers should capitalize on pupils' correct ideas. For example, pupils are correct in saying that something about the battery is constant. Teachers can say to pupils that something that is constant is called 'voltage' and not 'current'. This strategy is known as concept substitution.

Another example on the application of concept substitution is when pupils need to reason why the bulbs in series are all of the same brightness. The teacher can work from the notion that pupils hold namely that something is used up in the circuit. However, in this case something that is used up is electrical energy and not current. The depletion of electrical energy results in the battery to going 'flat'.

The use of mechanical analogies such as a bicycle chain is said to help understanding circuit. Figure 7 describes how a bicycle chain is analogous to a simple circuit. Figure 7 is taken from Summers, Kruger & Mant (1995) entitled 'The Primary School Teachers and Science (PSTS) Project'.

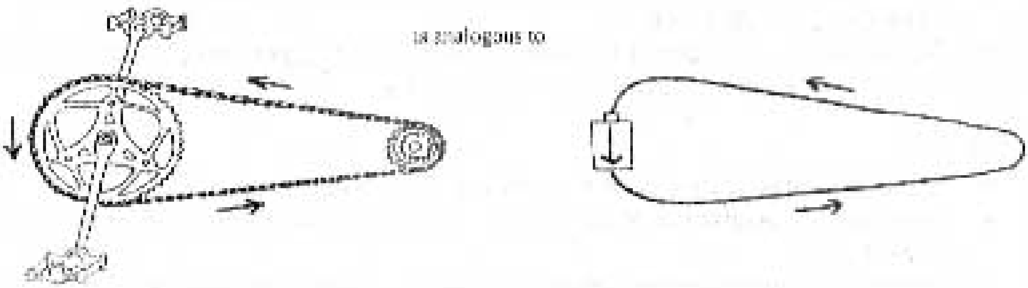


Figure 7 : The bicycle chain

Characteristics of the bicycle chain		**Characteristics of the circuit**
Links of the bicycle	Are analogous to	The electrons
Moving pedals	Are analogous to	The battery
Links are already at all points around the chain before the pedals start moving. Thus the pedals do not create the links.	Are analogous to	All electrons are ready in the wires before the battery is connected. Thus battery does not create electrons.

1.6 Conclusion

This paper argues that teachers need to realize pupils' ideas that they bring to the classroom and the need to take into account these ideas in their teaching so that learning can be meaningful and lasting for the pupils. We also suggested a few techniques that teachers can use to probe pupils' thinking and misconceptions. There are advantages and disadvantages in each technique. However, as suggested by Grayson (1997) asking the pupils to write their prediction, which is done mostly in POE activities, is an effective way to identify pupils' view about the concepts. It also makes them commit themselves to taking a stand of the views they put forth.

Teachers also need to be aware that probing pupils' thinking is not limited to the starting point of the teaching. It should be an integral and ongoing part of the science classroom activity. A teacher can employ any of the activities suggested earlier or through more careful questioning and answering activities.

Lastly, teachers who are new to these techniques should not expect them to work perfectly for the first time. It requires the cooperation of both the teachers and pupils. As a user, the teacher needs to be skillful and experienced in probing pupils' thinking while the pupils' as learners need to learn how to respond to such activity. Hence with time and practice both teachers and pupils are able to benefit from a potentially effective teaching and learning environment.

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