# THE REGULAR AND PART-TIME MANPOWER PLANNING FOR A FIRM TO OFFER INFORMATION SERVICE 

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#### Abstract

The objectives of this paper are to explore the issues regarding applying regular workers and part-time workers on customer information services via a telecommunication network, to formulate relevant costs to analyze their relationships with the level of customer services under the time dependent demand pattern, and to construct a manpower requirement model. Moreover, a heuristics algorithm is developed to solve the problem and an example is performed to illustrate the application of the model. The result shows that the manpower requirement at peak hour and the total cost paid by the firm for customer information service can be cut down by using different combinations of regular workers and part-time telecommuters, though a small amount of service delays will occur.


## INTRODUCTION

Today, the market condition faced by a firm is very complex and the level of information service requested by customers raise sharply. Many firms focus on their core business by changing organization structure so as to reduce unnecessary costs and put themselves in an advantageous position. Furthermore, people can communicate each other easily via a variety of mediums due to the advanced information technology. Accordingly, there are more service channels via which the firm can provide information services to its customers, such as telephone service, internet service, service by part-time workers or telecommuters, etc.

In practice, it is difficult for a firm to schedule the number of regular employees in different time period so as to satisfy its customers' information demand which are usually with a dynamic pattern. The personnel cost will increase rapidly and there will be many idle employees in the off-peak period if the firm uses the peak-hour demand as a base to determine its manpower requirement. On the other hand, the service quality during the peak period will deteriorate if the firm uses the off-peak demand as a base. Compared to large companies, small and medium firms hold less resources such as labor or capital, so that it is difficult for them to offer a sufficient number of employees to serve their customers. Because the internet technology nowadays has advanced up to support rapid interactions, using telecommuters who work at home to offer part-time services in peak period has provided an answer for the firm to solve this problem. The objectives of this paper are to explore the issues regarding applying regular workers and part-time workers on the customer information service, to formulate the relevant costs, to analyze their relationships with the level of customer information service in a dynamic demand pattern, and to construct a manpower requirement model.

Previous literature mostly focuses on the problem of establishing a work schedule that satisfies the given manpower and service requirements and complies with regulation and budget constraints. However, the scheduling problems considered in the literature were much more general, and none of them dealt with issues such as simultaneously determining the manpower and service demand leveling as concentrated in this paper. Here are some references that are deemed important as they describe methods or algorithms for the construction of efficient schedules in a particular area, such as airline crew scheduling in Etschmaier and Mathaisel (1985) and in Desaulniers et al. (1997), employee service scheduling in Bechtold and Showalter (1987), operator scheduling in Segal (1974), nurse scheduling in Arthur and Ravindran (1981), toll collectors in Byrne and Potts (1973), shift scheduling for banking operations in Davis and Reutzel (1981), and telephone sales manpower planning in Gaballa and Peace (1979). However, there were a few literature dealing with both the determination of manpower requirements and demand leveling, though none of them dealt with the issues regarding employing part-time telecommuters. Nobert and Jacques (1998) construct a freight handling personnel scheduling model at air cargo terminals for achieving cost reductions and determining the number of employees at different period while maintaining customer service levels.

The firms we focus here are those provide masses of information services or
those with a customer information service department. In this paper, the customer demand pattern is time dependent and with various service duration. Different degrees of urgency providing for the customer service are also considered. In this paper, we explore the relationship among the service level the firm offers for customer information inquiries and relevant costs paid for to achieve this service level. We use the difference in time between time expected by customers and time actually offered by firm to represent the time delay of customer information service and to represent the service level. Several types of time differences are analyzed by using different penalty costs to reflect their effects on the dissatisfaction or loss of customers. Mbreover, the relevant costs for the firm using various combinations of regular workers and part-time telecommuters to offer customer information service at a certain service level are considered. The costs we analyze include the salaries of regular workers and part-time telecommuters, rent, commuting costs and other personnel costs for regular workers, and information transferring costs and other costs for part-time telecommuters, etc. From the viewpoint of economic efficiency, we formulate penalty costs for various time delays of offering customer service and other relevant costs by analytical approach, and then construct a manpower requirement model. Finally, a heuristic algorithm is developed to solve the problem and an example is performed to illustrate the application of the model.

## MODEL FORMULATION

### 2.1 Basic Characteristics

The type of customer service we concern here is mainly information service. The customer request a service to the firm's service center through telecommunication network (such as telephone, fax, or internet), and the service center responses this request and provides the service via the same way. This is a common service type for many firms to setup their telephone service centers or internet consulting centers. Because the internet is available with lower and lower price today, more and more firms attempt to build their service centers in internet to satisfy more customers' inquiries. In addition, it is unnecessary for the firm's employees to commute between office and home due to no face-to-face service. Furthermore, the firm now may cut down total costs by using part-time telecommuters to offer a part of customer information services while maintaining the same service level.

The characteristics of the customers' information nquiries we consider here are with various time duration and different degrees of urgency. We explore the trade-offs between service levels and other relevant costs for firms using different combinations of regular workers and part-time telecommuters to provide their customers' information services, and then further analyze the manpower requirement at each time period. If the service request from the customer is urgent, the firm must make a response rapidly otherwise the firm may suffer the loss due to delay. On the other hand, if the request from the customer is not urgent, a bit of delay is acceptable.

We assumes that the firm pursues the goal of minimizing total costs including the penalty costs induced by the delays of offering services and other relevant costs paid by
the firm to offer customers' information service. The penalty cost is represented as reduced revenue owing to the dissatisfaction or loss of customers who are unsatisfied with the firm's information service response. Customer dissatisfaction comes from the discrepancy between the time customer expect to start or to finish the service and the time actually offered by a firm to start or to finish. Besides, the communication cost for customers waiting online for a service is also considered. Other relevant costs actually paid by firms include salaries for regular workers and part-time telecommuters, rent paid for the floor area of the information center, the commuting cost for regular workers, communication and transportation costs for part-time telecommuters, and the cost for equipment purchasing. The salaries of regular workers involve the health insurance premiums, retirement funds, and dismission funds. The rent paid for the floor area of the information center used by regular workers and part-time telecommuters is affected by the site of the center. The accessibility costs are defined as those paid by the firm's workers to contact with the information center or access to office. These costs are commuting costs for regular workers or transmission costs for telecommuters to transfer information or commuting costs for telecommuters to go back to office for meeting occasionally. The equipment for regular workers is different from that for part-time telecommuters, so the equipment purchasing cost will vary with the number of regular workers and the number of part-time telecommuters.

### 2.2 Cost Functions

In this section, we will analyze and formulate each of all costs paid by the firm for providing customer information services and explore interactions among them.

## 1. Penalty cost of service delays

We use the differences in time between the service starting time and the finishing time expected by customers and those actually offered by the firm to represent the level of service and define the $m$ as the delay of starting service and the delay of finishing service, respectively. The losses due to the delay of starting service or the delay of finishing service can be evaluated by giving various penalty costs for different types of delays. The concepts of delay are shown at figure 1. The two ends of white bar are the starting and finishing time expected by a customer to accept a service, respectively. And, the gray bar represents the length of service duration. There are three cases possible while the service is processed. If the firm immediately replys the service request of the customer, no delay will occur as shown by case 1 in figure 1 . If the firm can not reply the request immediately due to the shortage of available employees to dispatch, the delay of starting service arises as shown by case 2 and case 3 in figure 1 . In case 2 , there is no delay of finishing service even though there is a delay of starting service. In case 3, there is a delay of finishing service since the starting time actually offered by firm is too late.

Let $\mathrm{C}_{\mathrm{ijk}}$ be the number of customers who expect to start their services at $i$-th time interval and to finish at $j$-th time interval and the duration of processing the request is $k$ time intervals. Assume that if the firm actually starts the service at the time expected
to start by the customer, it is certain that the time at which the firm actually finishes this service will be before the time expected to finish by the customer, i.e., $\mathrm{i}+\mathrm{k} \leq \mathrm{j}$.

Let $\mathrm{x}_{\mathrm{ij}}^{\mathrm{t}}, \mathrm{t} \geq \mathrm{i}$ be the number of customers who can be counted to $\mathrm{c}_{\mathrm{ijk}}$ and actually start to be served by the firm at $t$-th time interval. Then the following equality is held.

$$
\begin{equation*}
\sum_{t} x_{i j k}^{t}=c_{i j k}, \quad \forall i j k \tag{1}
\end{equation*}
$$



Figure 1. The concepts of delays of starting service and finishing service
There is no delay of starting or finishing service for the customer group $x_{i j k}^{t}$, $\forall t=i$; while there will be some delay of starting service for the customer group $\mathrm{x}_{\mathrm{ik}}^{\mathrm{t}}$, $\forall t>i$, and the delay time is $(\mathrm{t}-\mathrm{i})$. Moreover, the total delay of starting service for customer group $\mathrm{C}_{\mathrm{ijk}}$ can be represented as the sum of the product of the number of customers who start to be served at each of different time intervals and their respective delay time, that is

$$
\begin{equation*}
\sum_{t}(t-i) x_{i j k}^{t} \tag{2}
\end{equation*}
$$

When the starting time at which the service is actually offered by the firm is so late as it affects on the finishing time of the service, the delay of finishing service may occur. In other words, if the actually finishing time $t+k$ for customer group $x_{i j k}^{t}$ is later than the time expected to finish the service by customers, the delay of finishing the service occurs and the delay time is $(t+k-j)$. On the other hand, if the actually finishing time $t+k$ is ahead of $j$, then there is no delay.

When there exists a delay of starting service or a delay of finishing service, customers will be unsatisfied with the service provided by the firm due to waiting. Those who receive the service with delay may look at the service unreliable or even suffer a loss for this reason. And then, it is possible that customers may give up the
service or choose another firm's service as the next information request arises. Assume the customer give-up rate is a probability and affected by the amount of the delay of starting service and the delay of finishing service. Let $p_{1}(h)$ and $p_{2}(h)$ as the give-up rates for the delay of starting service and the delay of finishing service as the delay time is $h$. And assume these two give-up rates are independent of each other. Then, the give-up rate for customer group $x_{i j k}^{t}$ is

$$
\begin{equation*}
P_{i j k}^{t}=p_{1}(t-i)+p_{2}(t+k-j)-p_{1}(t-i) p_{2}(t+k-j) \tag{3}
\end{equation*}
$$

and if $t+k<j, \quad p_{2}=0$.
Let the average potential revenue is $c$ dollars per service, then the penalty cost due to delay for customer group $x_{i j k}^{t}$ will be

$$
\begin{equation*}
P_{i j k}^{t} \cdot x_{i j k}^{t} \cdot c \tag{4}
\end{equation*}
$$

Though some of customers may still choose the services with delay, the firm's cash flow may be slightly postponed. However, since it is usually negligible and therefor could be neglected. Then, the total penalty cost for service delay is

$$
\begin{equation*}
c \sum_{i j k} \sum_{t} P_{i j k}^{t} \cdot x_{i j k}^{t} \tag{5}
\end{equation*}
$$

## 2. Communication Cost for Customers Waiting Online for Service

As the firm serves its customers through internet, the cost for information transferring must be paid and it equals to the product of the online time and the telecommunication fee per unit time. The customer online time for service includes the actually service duration $k$ and the time waiting online which equals the delay time of starting service. Let $u$ be the telecommunication fee per unit time, which is affected by the site of the firm, the condition of national information infrastructure and the distances from the site of the firm to those of customers. Then, the total communication cost for customers waiting online for service can be obtained as

$$
\begin{equation*}
\left[\sum_{\mathrm{ijk}} \mathrm{kx} \mathrm{x}_{\mathrm{ijk}}^{\mathrm{t}}+\sum_{\mathrm{ijk}} \sum_{\mathrm{t}}(\mathrm{t}-\mathrm{i}) \mathrm{x}_{\mathrm{ijk}}^{\mathrm{t}}\right] \mathrm{u} \tag{6}
\end{equation*}
$$

## 3. Salaries

The salaries offered by the firm for their regular workers and part-time telecommuters are usually different. The salaries for regular workers are mostly constant with fixed work hours, such as pay per month or per week, and there are extra benefits offered by the firm due to regulations, such as health insurance premiums, retirement funds, and dismission funds, etc. On the other hand, the salaries for part-time workers are related to their working hours and a little extra allowance is also included. Assume each service requested by a customer is served by one worker and only one. Then, it ought to be satisfied that the total number of workers at each time
interval is bigger or at least equals to the number of customers in service at the same time interval. The number of customers in service at time interval $t$, is equal to the sum of the number of customers who enter the service system at time interval $t$ and the number of all customers who enter the service system before $t$ but their requests have not yet been finished by $t$. Therefore, the total number of customers in service at interval $t$ is

$$
\begin{equation*}
\mathrm{x}^{\mathrm{t}}+\sum_{\mathrm{n}=1}^{\mathrm{t}-1} \sum_{\mathrm{k}>\mathrm{n}} \mathrm{x}_{\mathrm{k}}^{\mathrm{t}-\mathrm{n}}, \quad \forall t \tag{7}
\end{equation*}
$$

where

$$
\begin{aligned}
& x^{t}=\sum_{i j k} x_{i j k}^{t}, \quad \forall t \\
& x_{k}^{t}=\sum_{i j} x_{i j k}^{t}, \quad \forall t k
\end{aligned}
$$

Denote $P_{s 1}$ and $P_{s 2}$ as the periods of the time covered respectively by a regular-working shift $s 1$ and by a part-time working shift $s 2$. Let $\mathrm{e}_{r}^{\text {sl }}$ be the number of regular workers working at the regular-working shift $s 1$, and $\mathrm{e}_{\mathrm{p}}^{\mathrm{s2}}$ be the number of part-time workers working at the part-time-working shift $s 2$. Then, the total number of regular workers and part-time telecommuters at interval $t$ should be greater than or equal to the total number of customer in service at interval $t$, for all time intervals, as shown by constrains below:

$$
\begin{equation*}
\sum_{s 1: t \in e_{\mathrm{s}}} \mathrm{e}_{\mathrm{r}}^{\mathrm{sl}}+\sum_{\mathrm{s} 2: \mathrm{t} \in \mathrm{P}_{\mathrm{p}}} \mathrm{e}_{\mathrm{s} 2} \geq \mathrm{x}^{\mathrm{t}}+\sum_{\mathrm{n}=1}^{\mathrm{t}-1} \sum_{\mathrm{k}>\mathrm{n}} \mathrm{x}_{\mathrm{k}}^{\mathrm{t}-\mathrm{n}}, \quad \forall \mathrm{t} \tag{8}
\end{equation*}
$$

In general, the salaries paid by the firm for regular workers are related to the number of regular workers employed by firm. Thus, it is important for the firm to know the required number of regular workers to serve and fulfill the customers' information requests at each time interval. The total person-shifts for the total planning time $T$ can be determined by the definition of $\mathrm{e}_{\mathrm{r}}^{\mathrm{sl}}$ as $\sum \mathrm{e}_{\mathrm{r}}^{\mathrm{sl}}$. Let $S$ be the average number of shifts served by a regular worker, then we cansi ${ }^{2} \bar{b} b$ tain the number of regular workers needed by firm is

$$
\begin{equation*}
\frac{\sum_{s 1} e_{r}^{s 1}}{S} \tag{9}
\end{equation*}
$$

However, if the number of customers' information requests in different time interval varies and appears in a peaking pattern, then the required number of workers needed by firm to serve customers in different shifts will also vary and exhibit the same peaking phenomenobener ${ }_{r}^{\text {sl }}$ The maximal number of regular workers the firm can use at any time intervals is $\frac{\mathrm{sl}}{\mathrm{S}}$ as shown in equation (9). If the required number of regular workers needed by firm at each $\langle$ feill time intervals is smaller than the maximal number, then the firm can hire merely $\frac{s_{1}}{\varsigma}$ regular workers to serve their customers. If the number of regular workers needed at some time intervals exceeds this maximal number,
the firm may hire the number of workers based on the maximal required number of workers among for all shifts. Briefly, the total required number of regular workers needed by firm for customer information service is

$$
\begin{equation*}
E_{r}=\max \left[\max _{s 1}\left(e_{r}^{s 1}\right), \frac{\sum_{s 1} e_{r}^{s 1}}{S}\right] \tag{10}
\end{equation*}
$$

Let the average wage per regular worker be $w$, and the health insurance premiums, retirement funds and dismission funds which are extra benefits offered by the firm is a fraction of $w$. Denote this fraction by $r$, then the total salaries for all regular workers are

$$
\begin{equation*}
E_{r}(1+r) w \tag{11}
\end{equation*}
$$

The salaries for part-time telecommuters are usually related to their actual working hours. In general, the firm must promise in the contract that part-time workers at least have a certain lower bound of working hours with a wage rate larger than the minimum wage.. Part-time telecommuters who bear less workload still get a certain amount of the minimal pay. On the other hand, the lower bound of working hours is an obligation of part-time telecommuters that ensures the firm can obtain reasonable labor hours. Assume both the lower bound of working hours and the minimal wage are satisfied, i.e., the working hours per part-time telecommuter are all greater than the lower bound of working hours and the wages are all greater than the minimal wage. Then, we can determine the total salaries for all part-time telecommuters by multiplying the total working hours served by telecommuters by the average wage rate. Let the average wage rate for $s 2$ is $w_{p}^{s 2}$, then the total salaries for part-time telecommuters are

$$
\begin{equation*}
\sum_{s 2} e_{p}^{s 2} w_{p}^{s 2} \tag{12}
\end{equation*}
$$

## 4. Rent for the information service center

The regular workers we discuss here are those who commute to office to work, and the part-time telecommuters are those who work at home. In general, the firm will not subsidy the rent of floor area used at home by part-time telecommuters. Therefore, the rent for the information service center will be mainly affected by the number of regular workers. Denote the average size of floor area per regular worker by $\alpha_{1}$, and the rent for the service center sited at $j$ by $\mathrm{p}_{\mathrm{j}}$. Assume that the shifts of regular workers do not overlap each other, and the same office equipment and floor area allows to be used alternately by different regular workers working in different shifts. So, we can set the maximal number of regular workers among all shifts as the planning capacity, and the rent for the service center used by regular workers will be

$$
\begin{equation*}
\alpha_{1} p_{j} \max \left(e_{r}^{s 1}\right) \tag{13}
\end{equation*}
$$

Though the part-time workers work at home, but they need to go back to office for training, re-training, meeting, or other activities occasionally. The frequency for these
activities is low, but the firm still needs to prepare some space for them. Assume each part-time telecommuter must return to office every $n$ shifts, and all part-time telecommuters return to office by $\beta$ batches. Then, the number of part-time telecommuters return to office per batch is

$$
\begin{equation*}
\frac{\sum_{\mathrm{s} 2} e_{\mathrm{p}}^{\mathrm{s} 2}}{\mathrm{n} \beta} \tag{14}
\end{equation*}
$$

Let the average floor area used by each returned telecommuter is $\alpha_{2}$, then the rent for the service center used by part-time telecommuters is

$$
\begin{equation*}
\alpha_{2} p_{j} \frac{\sum_{s_{2}} e_{p}^{s_{2}^{2}}}{n \beta} \tag{15}
\end{equation*}
$$

## 5. Accessibility cost

We define the accessibility cost as the time cost and monetary cost paid by workers for commuting or telecommuting to office. For regular workers, the costs are commuting costs between office and home. For part-time workers, the costs are telecommunication expense used for communicating with the information center and the transportation cost for returning to office periodically. The accessibility cost includes both the monetary cost and time cost for regular workers and part-time telecommuters. The time cost reflects the opportunity cost of the time spent on commuting or telecommuting and is measured by the value of time which is assumed to be equal to the wage per hour. The wage rate is different between regular workers and part-time telecommuters due to the different ways used to determine their salaries. For each regular worker, the average wage rate can be represented as the total salary at planning period divided by the total working hours, that is

$$
\begin{equation*}
\frac{w}{\sum_{s \in \mathbf{S}} t\left(P_{s 1}\right)} \tag{16}
\end{equation*}
$$

where $t\left(P_{s 1}\right)$ : the length of time covered by one regular shift $s 1$;

S: the set of all shits offered per regular worker.
The average wage rate for part-time telecommuters can be represented as the average salary for each shift divided by the average length of time in one part-time shift, that is

$$
\begin{equation*}
\frac{\sum_{s 2} w_{p}^{s 2} / s 2}{\sum_{s 2} t\left(P_{s 2}\right) / s 2}=\sum_{s 2} \frac{w_{p}^{s 2}}{t\left(P_{s 2}\right)} \tag{17}
\end{equation*}
$$

where $t\left(\mathrm{P}_{\mathrm{s} 2}\right)$ : the length of time covered by one part-time shift $s 2$.

Assume the length of time in one regular shift is long enough so that it is impossible for a regular worker to work on two continuous shift, i.e. the regular worker ought to commute one round trip per shift. Let the average commuting distance of a round trip per regular worker is $\mathrm{d}_{1}$, and the travel speed is $v$. As each regular worker works on $S$ shifts, the total time cost for all regular workers used for commuting to office will be

$$
\begin{equation*}
\mathrm{E}_{\mathrm{r}} \mathrm{~S}\left(\frac{\mathrm{~d}_{1}}{\mathrm{~V}} \frac{\mathrm{w}}{\sum_{\mathrm{sl} 1 \mathrm{~s}} \mathrm{t}\left(\mathrm{p}_{\mathrm{sl}}\right)}\right) \tag{18}
\end{equation*}
$$

where $d_{1} / v$ is the time spent for one round trip. Let the unit-distance transportation fee is $f$. Then, the monetary cost paid by all regular workers commuting to office will be

$$
\begin{equation*}
\mathrm{E}_{\mathrm{r}} \mathrm{~S}\left(\mathrm{fd}_{1}\right) \tag{19}
\end{equation*}
$$

where $\mathrm{fd}_{1}$ is the monetary cost spent for one round trip. Then, the total accessibility cost for regular workers is the sum of the time cost and the monetary cost used for commuting to office, that is

$$
\begin{equation*}
E_{r} S\left(\frac{d_{1}}{V} \frac{w}{\sum_{s l \in s} t\left(p_{s l}\right)}+\mathrm{fd}_{1}\right) \tag{20}
\end{equation*}
$$

Because the part-time telecommuters do the job at home via telecommunication, the access time equals to the total time spent for communicating with office on line, i.e., it equals to the length of time covered by the total number of part-time shifts, so the total time for all part-time telecommuters access to office via telecommunication will be

$$
\begin{equation*}
\sum_{\mathrm{s} 2} \mathrm{e}_{\mathrm{p}}^{\mathrm{s} 2} \mathrm{t}\left(\mathrm{P}_{\mathrm{s} 2}\right) \tag{21}
\end{equation*}
$$

Denote $u$ as the unit-time fee for telecommunication, then the total monetary cost for part-time telecommuters used for information transferring is

$$
\begin{equation*}
\mathrm{u} \sum_{\mathrm{s} 2} \mathrm{e}_{\mathrm{p}}^{\mathrm{s} \mathrm{t}} \mathrm{t}\left(\mathrm{P}_{\mathrm{s} 2}\right) \tag{22}
\end{equation*}
$$

It must be noted, the accessibility cost for part-time telecommuter via telecommunication only includes monetary cost but not includes time cost. The part of time cost is paid as the salary for part-time telecommuters, therefore it cannot be computed for avoiding double counting.

As described above, part-time telecommuters must return to office for specified activities periodically, so the transportation cost for them should be counted. Let the average round-trip distance for a part-time telecommuter traveling to office once is $d_{2}$,
and the value of time is measured by the wage rate. $e^{52}$ use each part-time telecommuter must return to office every $n$ shifts, there are $s_{s 2}$ round trips made by all part-time workers for returning to office. Similar to equation (20), the total transportation cost for part-time telecommuters can be represented as

$$
\begin{equation*}
\frac{\sum_{s 2} e_{p}^{s 2}}{n} \cdot\left(\frac{d_{2}}{v} \cdot \sum_{s 2} \frac{w_{p}^{s 2}}{t\left(P_{s 2}\right)}+d_{2} f\right) \tag{23}
\end{equation*}
$$

## 6. The cost for equipment purchasing

Some communicating equipment is needed for the firm to serve their customers, such as central process system, servers or personal computers, etc. The equipment cost can be divided into two parts: one is the fixed cost, the other is variable cost which is related to the number of workers. Because the regular workers are formal long-term employees in the organization, the firm usually provides them equipment. On the other hand, the firm generally does not provide part-time telecommuters the equipment, but instead give a small amount of subsidy relating to their actual working hours. Let the fixed cost for equipment is $A$, and the variable cost of equipment purchased for each regular worker is $\mathrm{m}^{\mathrm{r}}$. Then, the total equipment cost excluding the part for part-time telecommuters will be

$$
\begin{equation*}
A+m^{r} E_{r} \tag{24}
\end{equation*}
$$

where $E_{r}$ is the total required number of regular workers needed by firm for customer service as defined by equation (10).

To compare the equipment cost with other costs at the same time base, let the life of service in years for all equipment $\mathrm{t}_{\mathrm{g}}$ is $b$ and the interest rate is $g$. Then, the capital-recovery-factor is $a=\frac{g(1+g)}{(1+g)^{b}}$. Consequently, the equivalent equipment cost spent over the planning peribd $\left.\boldsymbol{q}^{\prime}\right)^{b}$ days is

$$
\begin{equation*}
a\left[A+m^{r} E_{r}\right] \frac{T}{365} \tag{25}
\end{equation*}
$$

As discussed above, the part-time telecommuter is not formal employee of the firm, and generally the firm will not provide them the equipment, but give a small amount of subsidy relating to their actual working hours. Let $\mathrm{m}^{\mathrm{p}}$ denote the subsidy per part-time shift, then the total amount of subsidy for part-time telecommuters' equipment is

$$
\begin{equation*}
\mathrm{m}^{\mathrm{p}} \sum_{\mathrm{s} 2} \mathrm{e}_{\mathrm{p}}^{\mathrm{s} 2} \tag{26}
\end{equation*}
$$

### 2.3 Manpower Requirement Model

We can formulate here the manpower requirement model by combining each of all cost functions and relevant constraints discussed in section 2.1. The mathematical programming problem for the model is formulated as

Min.

$$
\begin{align*}
& c \sum_{i j k} \sum_{t} P_{i j k}^{t} \cdot x_{i j k}^{t}+\sum_{i j k} \sum_{t}\left[k x_{i j k}^{t}+(t-i) x_{i j k}^{t}\right] u \\
& +E_{r}(1+r) w+\sum_{s 2} e_{p}^{s 2} w_{p}^{s 2} \\
& +\alpha_{1} p_{j} \max \left(e_{r}^{s 1}\right)+\alpha_{2} p_{j} \frac{\sum_{s 2} e_{p}^{s 2}}{n \beta}  \tag{27}\\
& +E_{r} S\left(\frac{d_{1}}{v} \frac{w}{\sum_{s 1 \operatorname{S}} t\left(p_{s 1}\right)}+f d_{1}\right)+u \sum_{s 2} e_{p}^{s 2} t\left(P_{s 2}\right)+\frac{\sum_{s 2} e_{p}^{s 2}}{n} \cdot\left(\frac{d_{2}}{v} \cdot \sum_{s 2} \frac{w_{p}^{s 2}}{t\left(P_{s 2}\right)}+d_{2} f\right) \\
& +a\left[A+m^{r} E_{r}\right] \frac{T}{365}+m^{p} \sum_{s 2} e_{p}^{s 2} \tag{28}
\end{align*}
$$

$$
x_{i j k}^{t}, e_{r}^{s 1}, e_{p}^{s 2}, E_{r} \text { are integers }
$$

The objective function in equation (17) is to minimize the total costs including the penalty costs for service delay, the communication costs for customers waiting online for service, rent for the information service center, salaries, and accessibility costs for regular workers and for part-time telecommuters, and the costs for equipment purchasing. Constraint (28) specifies that customers in each of all customer groups will be served while the time intervals at which customers are actually served may be different. Constraints (29), (30), and (31) ensure that the number of workers working at each time interval is at least equal to the number of customers in service at the same interval. Constraint (32) defines the probabilities of the give-up rates for the delays of starting services and finishing services which are independent of each other. Constraint (33) states the number of regular workers hired by the firm is large enough at each of all shifts. The objective function (27) can be rewritten as

Min.

$$
+E_{r}\left\{(1+r) w+S\left(\frac{d_{1}}{v} \frac{w}{\sum_{s l \mathbf{S}} t\left(p_{s 1}\right)}+f d_{1}\right)+\frac{\stackrel{\text { HRM }}{a m} T}{365}\right\}
$$


where $\alpha_{1} p_{j} \max \left(e_{r}^{s 1}\right)+\frac{a A T}{365}$

$$
\begin{aligned}
& c_{1 i j k}^{t}=c P_{i j k}^{t}+[k+(t-i)] u \\
& c_{2}=(1+r) w+S\left(\frac{d_{1}}{v} \frac{w}{\sum_{s 1 \in \mathbf{S}} t\left(p_{s 1}\right)}+f d_{1}\right) \frac{a m^{r} T}{365} \\
& c_{3_{p}}^{s 2}=w_{p}^{s 2}+\frac{\alpha_{2} p_{j}}{n \beta}+u t\left(P_{s 2}\right)+\frac{1}{n} \cdot\left(\frac{d_{2}}{v} \cdot \sum_{s 2} \frac{w_{p}^{s 2}}{t\left(P_{s 2}\right)}+d_{2} f\right)+m^{p}
\end{aligned}
$$

The last term of equation (35) is a constant, so it can be omitted. Consequently, the manpower model will be

Min. $\sum_{i j k} \sum_{t} c_{1 i j k}^{t} i_{i j k}^{t}+c_{2} E_{r}+\sum_{s 2} c_{3_{p}}^{s 2} e_{p}^{s 2}+\alpha_{1} p_{j} \max \left(e_{r}^{s 1}\right)$
S.t. $\quad \sum_{t} x_{i j k}^{t}=c_{i j k}, \quad \forall i j k$

$$
\begin{aligned}
& \sum_{s 1: i t P_{s 1}} e_{r}^{s 1}+\sum_{s 2: t \in P_{s 2}} e_{p}^{s 2} \geq x^{t}+\sum_{n=1}^{t-1} \sum_{k>n} x_{k}^{t-n} \\
& x^{t}=\sum_{i j k} x_{i j k}^{t} \\
& x_{k}^{t}=\sum_{i j} x_{i j k}^{t} \\
& P_{i j k}^{t}=p_{1}(t-i)+p_{2}(t+k-j)-p_{1}(t-i) p_{2}(t+k-j) \\
& E_{r}=\max \left[\max _{s 1}\left(e_{r}^{s 1}\right), \frac{\sum_{s 1} e_{r}^{s 1}}{S}\right]
\end{aligned}
$$

$x_{i j k}^{t}, e_{r}^{s 1}, e_{p}^{s 2}, E_{r}$ are integers

## ALGORITHM

The manpower requirement model we formulate here is a NP-hard problem and the integral condition in the model makes it a difficult problem to solve exactly in polynomial time. Therefore, we develop a heuristic algorithm for this problem via shift-by-shift basis. The Algorithm is described as follows:

Step 1. Count the number of customers being served at each interval under the condition
of no delay.
Assume all customers of the firm can be served at the time they expect, i.e., the equation below is true.

$$
\begin{aligned}
& x_{i j k}^{t}=c_{i j k}, \text { for all } t=i \\
& x_{i j k}^{t}=0, \text { for all } t \neq i
\end{aligned}
$$

Because the duration of processing some service requests is longer than one interval, the number of customers in service at each interval is the sum of the numbers of those who enter in system at this interval and those who enter before but not complete their service at $t$. That is

$$
x^{t}+\sum_{n=1}^{t-1} \sum_{k>n} x_{k}^{t-n}, \quad \forall t
$$

Step 2. Count the number of regular workers $e_{r}^{s 1}$ for each of all shifts in case that there is no delay and no part-time telecommuter. That is

$$
e_{r}^{s 1}=\max \left[x^{t}+\sum_{\substack{n=1 \\ t \in P_{S 1}}}^{t-1} \sum_{k>n} x_{k}^{t-n}\right], \forall s 1
$$

Determine the minimal number of regular workers $E_{r}$ which should be hired by firm by:

$$
E_{r}=\max \left[\max _{s 1}\left(e_{r}^{s 1}\right), \frac{\sum_{s 1} e_{r}^{s 1}}{S}\right]
$$

Solve an initial solution via Step 1 and Step 2 and compute the total cost under the condition of no delay and no part-time telecommuter.

Step 3. Construct a manpower requirement submodel based on the length of time of one regular shift.

Step 3.1. Construct a submodel of $s 1$-th regular shift, and set the number of regular workers at the $s 1$-th shift, the number of part-time telecommuters at the corresponding simultaneous shifts, and the number of customers actually served at the $s 1$-th shifts as decision variables, and set other variables as given.

Then, relax the integral condition in this model and solve the corresponding LP problem. The form of submodel for the $s 1$-th regular shift is

$$
\text { Min. } \sum_{i j k} \sum_{t \in P_{s 1}} c_{1 i j k}^{t} t_{i j k}^{t}+c_{2} E_{r}+\sum_{s 2 \in P_{s 1}} c_{3}{ }_{3}^{s 2} e_{p}^{s 2}+\alpha_{1} p_{j} \max \left(e_{r}^{s 1}\right)
$$

S.t. $\quad \sum_{t \in P_{s 1}} x_{i j k}^{t}=c_{i j k}, \quad \forall i j k$

$$
e_{r}^{s 1}+\sum_{s 2 t \in P_{s 2}} e_{p}^{s 2} \geq x^{t}+\sum_{n=1}^{t-1} \sum_{k>n} x_{k}^{t-n}, \quad \forall t \in P_{s 1}
$$

$$
E_{r}=\max \left[\max _{s 1}\left(e_{r}^{s 1}\right), \frac{\sum_{s 1} e_{r}^{s 1}}{S}\right]
$$

$x_{i j k}^{t}, e_{r}^{s 1}, e_{p}^{s 2}, E_{r}$ are non-negatives

Step 3.2. Update the initial solution obtained in Step 2 by the result of the submodel of $s 1$-th regular shift. Then, return to Step 3.1 to construct and solve the submodel of the $(s 1+1)$-th regular shift.

Step 3.3 Repeat Steps 3.1 and 3.2, until submodels for all regular shifts in the planning period $T$ are solved.

Step 4. Round up the number of regular workers and the number of part-time telecommuters at each of all shifts and round up or down of the numbers of customers in service at each of all time intervals to obtain the solution. Compute the total cost for this solution at the end.

## EXPERIMENTAL RESULTS

We use an example to illustrate the application of the manpower requirement model formulated in this paper. The study planing period is a week. The length of time per regular shift is 8 consecutive hours and that per part-time shift is 4 consecutive hours. Therefore, there are 3 regular shifts and 6 part-time shifts in a day. The beginning times of regular shifts are $0 \mathrm{H}, 8 \mathrm{H}$, and 16 H , and the average salary per regular worker is 10000 NT dollars per week. The beginning time of the part-time shifts are $0 \mathrm{H}, 4 \mathrm{H}, 8 \mathrm{H}, 12 \mathrm{H}, 16 \mathrm{H}$, and 20 H . The average salary for the part-time telecommuter with shift beginning at 0 H or 4 H is 1000 NT dollars per shift, that for the part-time telecommuter with the shift beginning at 8 H or 12 H is 600 NT dollars, and that for part-time telecommuter with the shift beginning at 16 H or 20 H is 800 NT dollars. In this example, we set 10 minutes as a time interval, so there are 1008 time intervals during the planning period. All customers are categorized according to the degrees of urgency and the duration of processing their information requests. Regarding the duration of processing the request, all customers are divided into two groups. One is those whose requests can be finished at the duration of single time interval ( $k=1$ ), and the other is those whose requests are finished at the duration of two time intervals $(k=2)$. There are also two customer groups with regarding to the degrees of urgency. One is those whose requests are urgent and the firm must provide their requests immediately, and the other is those whose requests are not urgent and a bit of delay is acceptable. The give-up rates for the customer group whose information requests are urgent are

$$
p_{1}(t-i)=\left\{\begin{array}{cl}
0 & , t-i=0 \\
0.04 & , t-i=1 \\
0.16 & , t-i=2 \\
0.36 & , t-i=3 \\
0.64 & , t-i=4
\end{array} \quad p_{2}(t+k-i)=\left\{\begin{array}{cl}
0 & , t+k-i=0 \\
0.04 & , t+k-i=1 \\
0.16 & , t+k-i=2 \\
0.36 & , t+k-i=3 \\
0.64 & , t+k-i=4
\end{array}\right.\right.
$$

And the give-up rates for the customer group whose information requests are not urgent are

$$
p_{1}(t-i)=\left\{\begin{array}{cl}
0 & , t-i=0 \\
0 & , t-i=1 \\
0.04 & , t-i=2, \\
0.16 & , t-i=3 \\
0.36 & , t-i=4
\end{array} \quad p_{2}(t+k-i)=\left\{\begin{array}{cl}
0 & , t+k-i=0 \\
0 & , t+k-i=1 \\
0.04 & , t+k-i=2 \\
0.16 & , t+k-i=3 \\
0.36 & , t+k-i=4
\end{array}\right.\right.
$$

Assume the numbers of customers who is inquiring information in each time interval are distributed in a poisson distributions, then we can simulation the demand data by assuming the appropriate values for parameters. For customers whose requests can be finished by a single time interval, we assume the parameters of arrival rates for those whose requests are urgent and not urgent as 5 and 6 person/interval, respectively. For customers whose requests can be finished by two time intervals, we set the parameters of arrival rates for those whose requests are urgent and not urgent as 5 and 6 person/interval, respectively. Other parameters for the model are assumed by inferring and referring from the data in the reality and shown in Table 1.

Figure 2 shows the number of customers in service at each time interval with no delay and no part-time telecommuter hired. The total person-shifts of required regular shifts is shown to be 969 person-shifts, and the number of regular workers must be hired by firm is 162 workers when the average number of shifts served by a regular worker is 6 shifts during the study period. And, the corresponding total cost paid by the firm to provide the customer information services during the study period is $2,270,290$ NT dollars.

Figure 3 shows the number of customers in service at each time interval by applying the manpower requirement model we develop in the paper. The differences

Table 1. The values of parameters used for the model

| Variable | Description | Value |
| :---: | :--- | :---: |
| $c$ | The average potential revenue per service (NT dollar) | 100 |
| $u$ | The telecommunication fee per unit time (NT dollar) | 0.16 |
| $r$ | The fraction of wage for extra benefits offered by firm (\%) | $10 \%$ |
| $\alpha_{1}$ | The average size of floor area per regular worker (m²/worker) | 15 |
| $p_{j}$ | The rent for the service center sited at $j$ (NT dollar/ $\mathrm{m}^{2}$ ) | 11.635 |
| $\alpha_{2}$ | The average size of floor area used by each returned telecommuter <br> $\left(\mathrm{m}^{2} /\right.$ /worker $)$ | 5 |
| $n$ | The average number of shifts during which each part-time <br> telecommuter returning to office once | 10 |
| $\beta$ | The number of batches all part-time telecommuter return to office | 2 |
| $S$ | The average number of shifts a regular worker works on | 6 |
| $\mathrm{~d}_{1}$ | The average round-trip commuting distance per regular worker (km) | 20 |
| $d_{2}$ | The average round-trip distance for a part-time telecommuter <br> traveling to office once (km) | 25 |
| $v$ | The average travel speed (km/hr) | 38 |
| $f$ | The unit-distance transportation fee (NT dollar/km) | 1.56 |
| $a$ | The capitalrecovery-factor with the life of service in $b=5$ years and <br> the interest rate $g=0.06$. | 0.2374 |
| $A$ | The fixed cost for equipment (NT dollar) | 500,000 |
| $\mathrm{~m}^{r}$ | The variable cost of equipment purchased for each regular worker <br> (NT dollar) | 20,000 |
| $\mathrm{~m}^{\mathrm{p}}$ | The subsidy per part-time shift (NT dollar/shift) | 50 |



Figure 2. The number of customers in service at each time interval with no delay and no part-time telecommuters hired.


Figure 3. The number of customers in service at each time interval with allowing delays of services and hiring part-time telecommuters.
between the number of customers at peak interval and those in off-peak interval are shown to be markedly reduced as compared with those shown in Figure 2. The total person-shifts of regular shifts obtained by our model is 708 person-shifts and the number of regular workers must be hired by firm is 118 workers as the average number of shifts served by a regular worker is still 6 shifts during the study period. The total person-shifts of part-time shifts is 100 person-shifts and equivalent to about 50 person-shifts of regular shifts. The manpower requirement by allowing delays and hiring part-time telecommuters is shown to be cut down substantially. Consequently, the total cost paid by the firm for providing customer information services via employing both regular workers and part-time telecommuters as determined by the model is $2,270,290$ NT dollars. The saving is shown up to 661,400 NT dollars.

Regarding the delays of providing service, there are 6,674 customers served with delay, and the average delay is 1.024 intervals per delayed customer, while the average delay is only 0.311 intervals for all 21,976 customers. For those who are served with delay, $28.5 \%$ of them are with urgent requests, and $71.5 \%$ are with non-urgent requests. The result also shows that the manpower requirement at peak hour and the total cost paid by the firm can be cut down effectively by allowing delays of services and using different combinations of regular workers and part-time telecommuters, though a small amount of delays will occur to about one-third of total customers.

## CONCLUSION

This paper develops a model on the manpower requirement of customers' information services which are provided by firm via the telecommunication network. The demand pattern for information requests with various duration and different degrees of urgency is considered. The paper explores the relationships between the level of service the firm offering and the costs paid by firm to achieve this service level and then
formulate relevant cost functions and construct a manpower requirement model for the firm providing customers information services by employing different combinations of regular workers and part-time telecommuters. Furthermore, a heuristic algorithm is developed to solve the problem and an example is implemented to illustrate the applications of the model. The results show that using part-time telecommuters to offer a part of customer services will reduce both the problem of the worker shortage at peak hour and the problem of workers idle at off peak. Moreover, allowing a small amount of service delays will decrease the difference between the number of customers who requests a service at peak hour and those at off peak. These strategies provide an answer for the firms that are burdened with heavy expense on customer information services.

We suggest that several situations might be tested in the future via this model, such as the situation where the number of customers' requests is at each interval with a large variation, or the situations for the different sites of the information center. The effects of the changes in the values of the key variables in the model such as the level of service and relevant costs also can be examined by performing the sensitive analysis. Finally, the length of time interval for describing customers' arrivals and service duration also should be adjusted based on the characteristics of various services on different types of industries.

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